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Evaluation of Strategic Management and Growth Alternatives in Independent Auditing Firms Using Interval Type-2 Fuzzy ARAS Method

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ABSTRACT

Independent audit firms (IAFs) need to base their strategic management and growth decisions on more logical and rational foundations due to the technological and digital transformation, which has been increasing pressure in the recent period, the intensification of competition in the industry, and the rapid and frequently changing structure of legal regulations. In this context, this study aims to evaluate and prioritize strategic management and growth decisions systematically and analytically by identifying the strategic decision problems faced by IAFs. In this direction, six strategic alternatives determined by a comprehensive field study were analyzed with the help of the Interval Type-2 Fuzzy ARAS (IT2F–ARAS) method, considering thirteen criteria determined based on expert opinions. IT2F–ARAS technique has been preferred as it can effectively include differences in interpretation in specialists' judgments and ambiguities in the model. This procedure enables the integrated and flexible evaluation of quantitative and qualitative data in decision-making processes. When the results obtained by applying the proposed model are examined, it is seen that benefit-oriented criteria such as expanding the customer base, providing a competitive advantage, and increasing brand value are more decisive. It has been observed that cost-side criteria, such as initial investment cost and technological incompatibility, limit the applicability of some strategies. In addition, "digital transformation," "expanding the range of services," and "market diversification" have emerged as priority strategies for IAFs in the context of strategic management and growth decisions. The study aims to contribute to developing more consistent and sustainable growth policies by providing independent audit firms with a multi-criteria and fuzz-based decision support perspective in their strategic orientations.

1. Introduction

In addition to the disruptive technological developments that have emerged in the Industry 4.0 process, rapid and dynamically changing economic developments cause IAFs to face more

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complexities and uncertainties in their strategic decision-making processes than in the past. In addition to the digital and technological transformation that IAFs must carry out successfully, the increasing regulatory pressures of authoritarians, changes in customer expectations, and the increasing and intensifying competition in the sector make it necessary for IAFs to make long-term strategic decisions for sustainable growth as well as maintaining the quality of the services they produce. In that regard, evaluating strategic alternatives such as expanding the range of services, strengthening the digital infrastructure, opening to new markets, and talent management is vital in terms of the competitiveness and sectoral position of the companies. However, these strategic decisions must be treated as highly complex and highly affected by uncertainties, as they require considering many contradictory and interconnected criteria and different stakeholder perspectives.

In the context of IAFs, it may not be possible for strategic decision-making problems to be handled effectively and successfully and for practical and rational solutions to be produced with the intuitive experiences and biased attitudes of decision-makers. For this reason, it is vitally critical to address these decision problems using systematic, flexible decision support mechanisms that can manage existing uncertainties. On the other hand, according to the authors' information, there are no studies in the relevant literature in which the strategic growth decisions of IAFs are handled holistically and systematically within the framework of multi-criteria decision-making (MCDM) approaches using mathematical models. Existing studies in the literature have mainly focused on audit quality, regulatory compliance, or operational processes, and decision problems in the context of strategic management and growth have been mostly ignored. In addition, it is seen that traditional decision-making approaches, which do not adequately consider the differences in interpretation, uncertainties, and graded insecurities in expert judgments in such decision problems where uncertainty is high, are insufficient to provide a holistic, comprehensive and multidimensional roadmap to the decision-makers in the relevant industry. Accordingly, the relevant literature has critical and vitally important research gaps.

In this study, to eliminate this gap, the Interval Type-2 Fuzzy ARAS (IT2F–ARAS) method is proposed to evaluate the strategic management and growth alternatives of independent audit firms. This method produces more realistic and reliable results than traditional methods, thanks to its ability to flexibly model expert opinions in highly uncertain decision environments. With the IT2F-ARAS method, both benefit and cost-side criteria are handled within the same framework, and the uncertainties and differences in interpretation seen in expert evaluations are considered more accurately through the range representation. Thus, it holistically integrates numerical data and qualitative expert knowledge into decision processes. It is predicted that this method, which is used in limited numbers in the literature, will make an essential theoretical contribution in terms of multidimensional analysis of strategic trends in professional service sectors such as independent auditing.

As a result of the analyses carried out within the scope of the study, it was determined that beneficial criteria such as expanding the customer base, gaining competitive advantage, and increasing brand value were the determining factors in the decision process. On the other hand, it has been revealed that cost-oriented criteria such as initial investment cost, technological infrastructure incompatibility, and training needs restrict the applicability of some strategies. According to expert evaluations, digital transformation, expanding the range of services, and market diversification, among the alternatives, have become the top priority strategies. These findings guide decision-makers to balance between short-term and long-term goals and plan their corporate resources accordingly. In addition, the fact that the proposed model provides a systematic and structured evaluation framework in fuzzy multi-criteria decision-making environments will contribute to the strategic decision-making processes in the industry on more rational foundations.

It is considered that this study has made various contributions to both strategic management and multi-criteria decision-making literature. Presenting a model suitable for the fuzzy and multidimensional nature of strategic decisions and close to real-life conditions reveals a valuable approach in terms of both theoretical and practical application. It also fills a significant gap in creating a reference framework for future studies on strategic decisions in the independent audit sector.

The remaining sections of this article are structured as follows: In the second part, the relevant literature is reviewed, and research gaps are detailed. In the third part, the IT2F–ARAS method used in the study is explained, and the decision model is presented. The fourth section defines the implementation process, criteria, alternatives, and expert data. In the fifth chapter, the analysis results are shared and discussed. In the last part, managerial inferences are made in light of the findings, the study's limitations are discussed, and suggestions for future research are presented.

2. Research Background

Audit quality is a multidimensional concept that has been discussed in the accounting and finance literature for many years, and it has undergone a significant transformation over time in terms of theoretical and practical perspectives. This concept was primarily associated with the trustworthiness of financial reporting, the robustness of internal control systems, and the effectiveness of corporate governance mechanisms in previous investigations. Research between 2011 and 2020 focused on the determinants of structural governance elements such as board characteristics, ownership structures, and audit committee qualifications on audit quality. In this context, Loi et al. [1, 2] studies reveal the interaction between governance practices and audit processes by analyzing the effects of corporate governance principles on sustainability accounting and internal control systems. Similarly, in a review conducted by Wahhab and Khlaif [3] in the Iraqi banking sector, the effects of audit committee characteristics on financial reporting quality were evaluated. However, it was concluded that different types of ownership did not significantly affect this relationship.

In the relevant literature after 2021, a remarkable paradigm shift is striking. Audit and reporting quality is no longer limited to the technical competence or independence of the auditor. Still, it is associated with broader structural and behavioral elements such as customer relations, quality of service, decision-making processes, and digitalization. In the study conducted by Le Nhan et al. [4], the factors affecting the customer acceptance decisions of audit firms were systematically classified under six main headings, and how these factors are related to audit quality was detailed. In parallel, Nguyen et al. [5] examined the relationship between service quality, trust, customer satisfaction, and customer loyalty in the Vietnamese audit sector. They emphasized that the audit process is also a service delivery and customer management process. These findings go beyond the traditional audit approach and show that the quality of the relationship between the customer and the service experience is also decisive in the overall audit performance.

Another trend that has gained momentum in recent years is the holistic evaluation of audit activities with environmental, social, and governance (ESG) performances. In this context, Han [6] analyzed the impact of governance mechanisms on ESG practices and argued that auditing plays an assurance role. On the other hand, Pham et al. [6] evaluated audit firms' contributions to ESG performance in their study in Vietnam; however, they could not detect a significant quality difference between large audit firms (Big 4) and other companies in this context. The key determinants of audit quality may be more closely related to process-specific application quality, not just scale or brand. On the other hand, Jácome et al. [7], in their research conducted in Brazil, examined the systemic barriers to ethical reporting behaviors of managers and found that ethical responsibilities were not sufficiently internalized, emphasizing that auditing should not only be a technical process but also an ethical audit function.

The common findings of the above studies show that many internal and external factors play a role in determining audit quality. Auditor independence, level of professional expertise, service quality, audit fees, client onboarding process, reliability of internal control systems, and corporate governance practices stand out among these factors. A direct and substantial relationship exists between corporate governance principles and strategic accounting practices. Well-structured governance systems enable more reliable, transparent, and sustainable audit processes, directly reflected in the quality of financial reporting. At the same time, factors such as customer satisfaction, sense of trust, and service experience are decisive in the sustainability and competitiveness of audit firms in the sector. However, there is no consensus in the literature on the effect of ESG performance on audit quality. Han [8] argues that auditing contributes to ESG processes; Pham et al. [6] state that this relationship is weak. Such findings reveal the necessity of considering contextual variables in audit quality research.

Despite all these findings, it is also clear that there are some structural gaps in the literature. Considering the multidimensional and uncertain nature of the factors affecting audit quality, it is evident that the current research is primarily based on deterministic and classical statistical analysis. However, the nature of such decision processes includes subjective judgments, expert opinions, and fuzzy variables, necessitating the use of more flexible and decision-supporting methods. Currently, multi-criteria decision-making (MCDM) methods, especially fuzzy logic-based approaches, do not find enough space in the literature. This deficiency becomes even more evident when evaluated with some criterion sets that find application in the literature. For example, criteria such as customer onboarding processes, service quality, ethical behaviors, and ESG performance defined in studies such as Le Nhan et al. [4] and Pham et al. [6] can be integrated into decision-making models.

In this context, advanced MCDM models, such as the Interval Type-2 Fuzzy ARAS (IT2F-ARAS) method, can contribute innovatively to the audit quality literature thanks to their flexibility in modeling uncertainties. The IT2F-ARAS method is highly applicable in allowing both the determination of criterion weights under uncertainty and the ranking of alternatives according to effectiveness. Thus, evaluating audit firms or strategic action plans according to multidimensional quality criteria can be more objective and decision-maker-friendly. In addition, this method evaluates current audit performance and makes it possible to test future improvement scenarios analytically.

In conclusion, the literature on audit reporting and quality has significantly transformed its scope and orientation. A more holistic approach has replaced narrow definitions based on technical competence with multifaceted elements such as customer relations, service experience, and sustainability. However, despite this multidimensionality, current methodological approaches fail to model complexity adequately. In this context, uncertainty-sensitive MCDM methods such as IT2F-ARAS can fill gaps in the academic literature and provide practitioners with decision-support tools. Such approaches can measure audit quality and guide strategic improvement and organizational transformation processes.

3. Methodology

This section demonstrates the suggested decision-making model and its fundamental algorithm. The basic algorithm of the model is illustrated as follows:

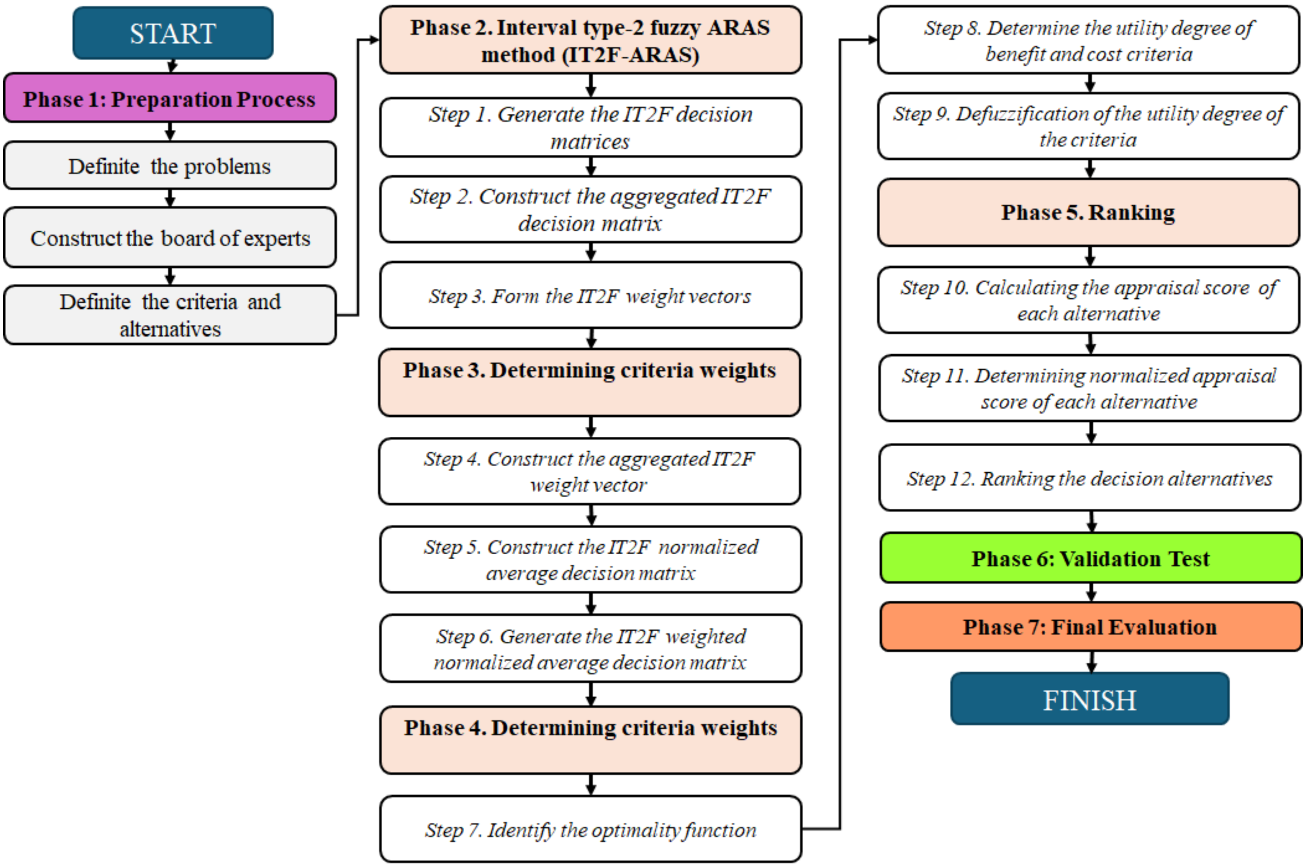

Fig. 1. The basic algorithm of the proposed IT2F-ARAS technique

3.1. Preliminary information on interval type-2 fuzzy sets

Type-2 fuzzy sets offer several advantages over their type-1 counterparts. One key benefit is their ability to manage higher levels of uncertainty, as their membership functions (μ_A) are not strictly limited to values between 0 and 1. Type-1 fuzzy sets are often criticized for failing to capture the inherent uncertainties within complex systems, which limits their effectiveness in ambiguous situations. To address these shortcomings, interval type-2 fuzzy sets extend type-1 fuzzy sets by incorporating additional layers of uncertainty. A type-2 fuzzy set A, along with its membership function (μ_A) defined over the universe of discourse X, can be expressed using the following Equation.

$$A=\left\{\left(\left(x,u\right),\mu_A\left(x,u\right)\right)\middle|\forall u\in J\subseteq[0,1],0\leq\mu_A\left(x,u\right)\leq 1\right\} \tag{1}$$

J_x depicts an interval [0,1], and the type-2 fuzzy set A can be exhibited as follows:

$$A=\int_{x\in X}\int_{u\in J_X}\frac{\mu_A\left(x,u\right)}{\left(x,u\right)} \tag{2}$$

$J_x \subseteq [0,1]$ and $\int\int$ illustrates union all over allowable x and u respectively. In an interval type-2 fuzzy set, the membership function $\mu_A(x,u)$ equals 1. An interval type-2 fuzzy set is an extended form of a Type-2 fuzzy set, and it can be described as follows:

$$A = \int_{x \in X} \int_{u \in J_X} \frac{1}{(x,u)} \tag{3}$$

An interval type-2 fuzzy set can be characterized through its primary membership function, which forms what is known as the footprint of uncertainty (FOU). This footprint represents the range of uncertainty in the membership grades. A trapezoidal interval type-2 fuzzy set is defined using this concept, as described by Karagöz et al. [9].

$$A = \left(A^U, A^L\right) = \left(\left(a_1^U, a_2^U, a_3^U, a_4^U; h_1\left(A^U\right), h_2\left(A^U\right)\right), \left(a_1^L, a_2^L, a_3^L, a_4^L; h_1\left(A^L\right), h_2\left(A^L\right)\right)\right), \tag{4}$$

Where, A^U and A^L denotes the type-1 fuzzy sets a_l^U and a_l^L, $l = 1,...,4$ symbolizes the reference points of the interval type-2 fuzzy set A. $h_k\left(A^U\right)$ depicts that the membership value of the element a_{k+1}^U in the upper trapezoidal fuzzy set $A^U, 1 \le k \le 2$; $h_k\left(A^L\right)$ symbolizes the membership value of the element a_{k+1}^L in the lower trapezoidal fuzzy set A^L [9–12].

3.2. Interval type-2 fuzzy ARAS method (IT2F-ARAS)

This study adopts the core steps of the ARAS methodology as originally proposed by Zavadskas and Turskis [13] while enhancing the approach by incorporating interval type-2 fuzzy sets, as developed in the work of Karagöz et al. [9].

The T2IF-ARAS method introduced in this study involves a sequence of 12 procedural steps, as outlined by Karagöz et al. [9] and Görçün et al. [14].

Step 1. Construct the IT2F decision matrices: At this stage, each decision-maker ($\rho = 1,2,...\tau$) provides linguistic assessments for every alternative $(i = 1,2,...m)$ relative to each criterion $(j = 1,2,...n)$, using the linguistic scale specified in Table 5. Based on these evaluations, the interval type-2 fuzzy (IT2F) decision matrices are formed accordingly.

$$\xi^{\rho} = \begin{bmatrix} \xi_{11}^{\rho} & \xi_{12}^{\rho} & . & \xi_{1n}^{\rho} \\ \xi_{21}^{\rho} & \xi_{22}^{\rho} & . & \xi_{2n}^{\rho} \\ . & . & . & . \\ \xi_{m1}^{\rho} & \xi_{m2}^{\rho} & . & \xi_{mn}^{\rho} \end{bmatrix} \tag{5}$$

Step 2. Form the aggregated IT2F average decision matrix: The individual decision matrices are combined using the arithmetic mean, as shown in the Equation below.

$$\xi_{ij} = \frac{\sum_{\rho=1}^{\tau} \xi_{ij}^{\rho}}{\tau} \tag{6}$$

Where ξ_{ij}^{ρ} depicts the relative significance value of i^{th} option concerning j^{th} criterion given by the ρ^{th} decision-maker. Also, τ symbolizes the number of decision-makers.

Step 3. Determine the IT2F weight vectors: In this phase, each decision-maker assesses the importance of each criterion using the predefined linguistic scale. These assessments are then transformed into interval type-2 fuzzy numbers (IT2FNs), generating multiple weight vectors τ as outlined below.

$$W^{\rho} = \begin{bmatrix} w_1^{\rho} \\ w_2^{\rho} \\ \cdot \\ w_n^{\rho} \end{bmatrix} \tag{7}$$

w_1^{ρ} denotes the evaluation performed by ρ^{th} decision-makers for j^{th} criterion.

Step 4. Generate the aggregated IT2F weight vector: The individual IT2F weight vectors are combined using Equation (8), following the aggregation procedure described by Deveci et al. [15].

$$w_j = \frac{\sum_{\rho=1}^{\tau} w_j^{\rho}}{\tau} \tag{8}$$

Step 5. Develop the IT2F normalized average decision matrix: Considering the nature of each criterion—whether it represents a cost or a benefit—the elements of the aggregated IT2F matrix are normalized using a linear normalization method, as detailed below.

$$\tilde{\eta}_{ij}^{*} = \left(\left(\frac{\xi_{ij1}^{U}}{\xi_{j*}^{U}}, \frac{\xi_{ij2}^{U}}{\xi_{j*}^{U}}, \frac{\xi_{ij3}^{U}}{\xi_{j*}^{U}}, \frac{\xi_{ij4}^{U}}{\xi_{j*}^{U}}; h_1\left(\xi_{ij}^{U}\right), h_2\left(\xi_{ij}^{U}\right) \right), \left(\frac{\xi_{ij1}^{L}}{\xi_{j*}^{U}}, \frac{\xi_{ij2}^{L}}{\xi_{j*}^{U}}, \frac{\xi_{ij3}^{L}}{\xi_{j*}^{U}}, \frac{\xi_{ij4}^{L}}{\xi_{j*}^{U}}; h_1\left(\xi_{ij}^{L}\right), h_2\left(\xi_{ij}^{L}\right) \right) \right) \tag{9}$$

$$\eta_{ij}^{-} = \left(\left(\frac{\xi_{j-}^{U}}{\xi_{ij4}^{U}}, \frac{\xi_{j-}^{U}}{\xi_{ij3}^{U}}, \frac{\xi_{j-}^{U}}{\xi_{ij2}^{U}}, \frac{\xi_{j-}^{U}}{\xi_{ij1}^{U}}; h_1\left(\xi_{ij}^{U}\right), h_2\left(\xi_{ij}^{U}\right) \right), \left(\frac{\xi_{j-}^{U}}{\xi_{ij4}^{L}}, \frac{\xi_{j-}^{U}}{\xi_{ij3}^{L}}, \frac{\xi_{j-}^{U}}{\xi_{ij2}^{L}}, \frac{\xi_{j-}^{U}}{\xi_{ij1}^{L}}; h_1\left(\xi_{ij}^{L}\right), h_2\left(\xi_{ij}^{L}\right) \right) \right) \tag{10}$$

Where $\tilde{\eta}_{ij}^{*}$ denotes the normalized values of benefit criteria and η_{ij}^{-} symbolizes the normalized values of cost criteria. Also, $\xi_{j*}^{U} = \max_{i=1,\dots,m,\ l=1,\dots,4} \xi_{ijl}^{U}$ for benefit criteria j and

$\xi_{j-}^{U} = \min_{i=1,\dots,m,\ l=1,\dots,4} \xi_{ijl}^{U}$ for a cost criteria j.

Step 6. Create the IT2F weighted normalized average decision matrix: The normalized IT2F matrix is assigned weights using Equation (11).

$$\delta_{ij} = w_j \otimes \eta_{ij} \tag{11}$$

δ_{ij} denotes the element value of each element of the normalized weighted IT2F matrix.

Step 7. Identify the optimality function of benefit criteria O_i^* and the optimality function of cost criteria O_i^- of each alternative:

$$O_i^* = \sum_{j \in B} \delta_{ij} \tag{12}$$

$$O_i^- = \sum_{j \in C} \delta_{ij} \tag{13}$$

Step 8. Calculate the utility degree for benefit and cost criteria: The utility degrees for both benefit and cost criteria are determined by applying Equations (14) and (15).

$$S_i^* = \frac{O_i^*}{\max_{i=1,\dots,m,\ l=1,\dots,4} O_{il}^*} \tag{14}$$

$$S_i^- = \frac{\min_{i=1,\dots,m,\ l=1,\dots,4} O_{il}^-}{O_i^-} \tag{15}$$

Step 9. Defuzzification of the utility degrees of the criteria: In this phase, we apply the defuzzification algorithm proposed by Chen et al. [16] and Karagöz et al. [9] to compute the defuzzified utility degrees for the criteria. The following Equation is used for this purpose.

$$\Theta_i^* = \left[\frac{s_{i1^*}^U + s_{i4^*}^U}{2} + \frac{h_1\left(S_{i^*}^U\right) + h_2\left(S_{i^*}^U\right) + h_1\left(S_{i^*}^L\right) + h_2\left(S_{i^*}^L\right)}{4}\right] \frac{\sum_{l=1}^{4}\left(s_{il^*}^U + s_{il^*}^L\right)}{8} \tag{16}$$

$$\Theta_i^- = \left[\frac{s_{i1-}^U + s_{i4-}^U}{2} + \frac{h_1\left(S_{i-}^U\right) + h_2\left(S_{i-}^U\right) + h_1\left(S_{i-}^L\right) + h_2\left(S_{i-}^L\right)}{4}\right] \frac{\sum_{l=1}^{4}\left(s_{il-}^U + s_{il-}^L\right)}{8} \tag{17}$$

Step 10. The appraisal score Δ_i of each variant is calculated by using Equation (18).

$$\Delta_i = \frac{\Theta_i^* + \Theta_i^-}{2} \tag{18}$$

Step 11. The normalized evaluation score for each alternative is determined using the following Equation.

$$\Omega_i = \frac{\Delta_i}{max_i\, \Delta_i} \tag{19}$$

Step 12. Ranking the decision alternatives: The alternatives are ranked based on their appraisal scores.

4. Case Study

This study applied the proposed model to address the decision-making problem concerning strategic management and growth alternatives for Independent Auditing Firms. Four experienced professionals were assembled in the initial phase to provide expert insights. The profiles of these experts are presented in Table 1.

Table 1
Details of the Experts

Code	Graduation Degree	Graduated Department	Experience	Expertise	Certifications
U1	Master	Business Administration (MBA)	15	Financial Analysis	CPA, CISA, IFRS
U2	Doctorate	Strategic Management	10	Strategic Planning	PMP, MBA,
U3	Master	Computer Engineering	8	Data Analytics	CISSP, ITIL,
U4	License	Human Resource Management	12	Training Programs	SHRM,
U5	Bachelor of Laws	Law	20	Legislation	Bar Certificate
U6	Master	Economy	14	Financial Analysis	CFA
U7	License	Marketing	10	Market Research	CRM

This investigation selected seven specialists to help set the criteria and strategy alternatives for assessing IAFs' strategic management and growth decisions. The practitioners aim to obtain more reliable and valid results, considering their experience and expertise in the sector. The experts have been selected from among people with deep knowledge and experience in the independent audit industry. It makes it possible for professionals who can better understand the developments in the sector and make more accurate decisions regarding strategic management to contribute.

The specialists are professionals who specialize in different fields in the industry. These areas include strategic management, digital transformation, market analysis, financial planning, and technological innovations. Each expert has determined the critical criteria of the study, considering their knowledge and sectoral experience in their field. Experts are knowledgeable about internal and external audit processes, and thanks to this, they understand strategic decisions by considering every aspect of audit firms. In addition, it is essential to select experts who can monitor changes in digital transformation, technological developments, and legal regulations in the sector. In this way, the strategic decision problems faced by independent audit firms have been better identified. Table 2 exhibits the strategy alternatives.

Table 2
The Strategy Alternatives

Codes	Strategies	Definitions
A1	Expanding Service Range	Increasing customer value by offering new services in non-regulatory areas (tax, finance, ESG).
A2	Expanding to New Markets	Diversification in terms of sectoral (healthcare, energy), geographical (international)
A3	Strategic Partnerships and Mergers	Merger/partnership with different stakeholders for competitive advantage, digitalization
A4	Developing Competitive Strategies	Developing competitive positions focused on cost, expertise, quality, or technology.
A5	Transforming Organizational Structure	Providing flexibility and growth capacity through structural changes (branch opening, franchise).
A6	Digital Transformation and Technology Investment	Increasing service quality and efficiency with technologies

Next, the researchers identified the criteria that must be handled for developing rational and reliable solutions, as shown in Table 3.

Table 3
The Selection Criteria

Code	Criterion Name	Explanation
C1	Expanding the Customer Base	Potential to reach new customer groups and expand the range of services.
C2	Contribution to Service Quality	Improve quality, reliability, and audit assurance in service processes.
C3	Brand and Reputation Increase	Potential to increase sectoral awareness, prestige, and customer confidence.
C4	Technology Infrastructure Incompatibility	Difficulty or incompatibility with existing digital infrastructure.
C5	Sustainability Risk	The risk is that the strategy cannot be sustained long-term and remain relevant.
C6	Technological Adaptation	The level of integration into new technologies, digital tools, and automation.
C7	Providing a Competitive Advantage	The power to differentiate against competitors and gain an advantage in the market.
C8	Creating New Revenue Streams	Ability to generate sustainable additional income from existing out-of-service areas.
C9	Initial Investment Cost	The initial size of the financial investment required to implement the new strategy.
C10	Training and Human Resource Requirements	The cost of training and personnel required for new skills and specializations.
C11	Operational Complexity	The level at which processes become more complex is an organizational strain.
C12	Application Time	The time required for the strategy to be fully implemented.
C13	Risk of Compliance with Regulations	Legal risks and compliance costs that may be encountered in compliance with legislation.

Here, we show the application of the proposed MCDM framework, which includes the IT2F-ARAS technique that was applied to solve the decision-making problem. The proposed methodological framework was used to determine the appropriate strategy alternative among the six strategic management and growth strategies identified. For this purpose, a set of selection criteria consisting of 13 factors has been determined, as shown in Table 3. Next, the decision-makers assessed the selection criteria using the linguistic terms in Table 4. The evaluations of the decision-makers are shown in Table 5.

Table 4
Linguistic terms and Interval Type-2 Fuzzy Sets (IT2FS) [9, 17, 18]

Linguistic Terms	**Abb.**	a_1^u	a_2^u	a_3^u	a_4^u	$h_1^u(A^U)$	$h_2^u(A^U)$	a_1^l	a_2^l	a_3^l	a_4^l	$h_1^l(A^L)$	$h_2^l(A^L)$
Very low	VL	0	0	0	0.1	1	1	0	0	0	0.05	0.9	0.9
Low	L	0	0.1	0.1	0.3	1	1	0.05	0.1	0.1	0.2	0.9	0.9
Medium low	ML	0.1	0.3	0.3	0.5	1	1	0.2	0.3	0.3	0.4	0.9	0.9
Medium	M	0.3	0.5	0.5	0.7	1	1	0.4	0.5	0.5	0.6	0.9	0.9
Medium high	MH	0.5	0.7	0.7	0.9	1	1	0.6	0.7	0.7	0.8	0.9	0.9
High	H	0.7	0.9	0.9	1	1	1	0.8	0.9	0.9	0.95	0.9	0.9
Very High	VH	0.9	1	1	1	1	1	0.95	1	1	1	0.9	0.9

Table 5
The decision-makers' linguistic appraisals for the criteria.

DMs	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
U1	H	VH	H	H	M	VH	MH	L	M	ML	M	L	MH
U2	VH	H	H	VH	H	H	VH	ML	H	M	MH	MH	L
U3	M	M	M	ML	VH	M	M	H	L	M	ML	L	VH
U4	H	M	M	H	ML	M	MH	L	VH	M	H	ML	L
U5	L	ML	L	L	L	ML	L	VH	M	H	M	VH	M
U6	M	H	MH	M	M	VH	H	M	M	ML	MH	L	L
U7	VH	M	MH	MH	ML	M	MH	H	M	ML	M	L	H

The linguistic assessments provided by the decision-makers were subsequently converted into Interval Type-2 Fuzzy sets (IT2Fs) based on the linguistic scale outlined in Table 5. Following this, the IT2F weights for each criterion were calculated using Equation (8), as illustrated in Table 6.

Table 6
IT2F weights of each criterion

	Average IT2F weights											
	W^U						W^L					
	$Wj1^u$	$Wj2^u$	$Wj3^u$	$Wj4^u$	$h_1(Wj)^u$	$h_2(Wj)^u$	$Wj1^l$	$Wj2^l$	$Wj3^l$	$Wj4^l$	$h_1(Wj)^l$	$h_2(Wj)^l$
C1	0.54	0.70	0.70	0.81	1	1	0.62	0.70	0.70	0.76	0.9	0.9
C2	0.47	0.66	0.66	0.80	1	1	0.56	0.66	0.66	0.73	0.9	0.9
C3	0.43	0.61	0.61	0.79	1	1	0.52	0.61	0.61	0.70	0.9	0.9
C4	0.46	0.63	0.63	0.77	1	1	0.54	0.63	0.63	0.70	0.9	0.9
C5	0.34	0.51	0.51	0.67	1	1	0.43	0.51	0.51	0.59	0.9	0.9
C6	0.50	0.67	0.67	0.80	1	1	0.59	0.67	0.67	0.74	0.9	0.9
C7	0.49	0.66	0.66	0.81	1	1	0.57	0.66	0.66	0.74	0.9	0.9
C8	0.39	0.54	0.54	0.69	1	1	0.46	0.54	0.54	0.61	0.9	0.9
C9	0.40	0.57	0.57	0.73	1	1	0.49	0.57	0.57	0.65	0.9	0.9
C10	0.27	0.47	0.47	0.66	1	1	0.37	0.47	0.47	0.56	0.9	0.9
C11	0.39	0.59	0.59	0.77	1	1	0.49	0.59	0.59	0.68	0.9	0.9
C12	0.21	0.34	0.34	0.51	1	1	0.28	0.34	0.34	0.43	0.9	0.9
C13	0.34	0.49	0.49	0.64	1	1	0.41	0.49	0.49	0.56	0.9	0.9

For instance, the IT2F weight for the C1 criterion—assessed using the linguistic terms (H, VH, M, H, L, M, VH)—is calculated using the following procedure.

$$\tilde{\tilde{W}}_1 = \sum_{\rho=1}^{7}\left[\frac{\left[(0.7,0.9,0.9,1,1,1),(0.8,0.9,0.9,0.95,0.9,0.9)\right]}{7}+....+\frac{\left[(0.9,1,1,1,1,1),(0.95,1,1,1,0.9,0.9)\right]}{7}\right]$$

$$=\left[(0.54,0.70,0.70,0.81,1,1),(0.62,0.70,0.70,0.76,0.90,0.90)\right]$$

Then, the defuzzified criteria weights were calculated, and the results are presented in Table 7 below.

Table 7
Defuzzied criteria weights

Code	Criteria	DEFUZZY	NORM.	RANK
C1	Expanding the Customer Base	1.127	0.100	1
C2	Contribution to Service Quality	1.029	0.091	4
C3	Brand and Reputation Increase	0.952	0.084	6
C4	Technology Infrastructure Incompatibility	0.975	0.086	5
C5	Sustainability Risk	0.745	0.066	10
C6	Technological Adaptation	1.061	0.094	2
C7	Providing a Competitive Advantage	1.047	0.092	3
C8	Creating New Revenue Streams	0.803	0.071	9
C9	Initial Investment Cost	0.861	0.076	8
C10	Training and Human Resource Requirements	0.663	0.059	12
C11	Operational Complexity	0.891	0.079	7
C12	Application Time	0.461	0.041	13
C13	Risk of Compliance with Regulations	0.705	0.062	11

The defuzzified weight of the C1 criterion, shown in Table 7, is derived by applying the defuzzification process to its corresponding IT2F value, as provided in Table 3.

$$Defuzzy_1 = \left[\frac{0.54+0,70}{2}+\frac{1+1+0.9+0.9}{4}\right]\otimes\left[\frac{(0.54+0.70+0.70+0.81+0.62+0.70+0.70+0.76)}{8}\right]=1.127$$

$$W_{n1}=\frac{1.127}{1.127+1.029+...+0.705}=0.100$$

Once the criteria weights were determined, the decision alternatives were assessed using the linguistic terms listed in Table 9. These linguistic evaluations are summarized in Table 8. In the next step, the evaluations were transformed into Interval Type-2 Fuzzy Numbers (IT2FNs) based on the corresponding linguistic evaluation scale.

Table 8
Linguistic evaluations for decision alternatives

	DMs	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
	DM1	VG	G	G	F	F	VG	G	P	F	F	F	MP	G
A1	**DM2**	F	MG	F	G	G	F	F	P	F	F	F	P	F
	DM3	VG	MG	G	F	F	VG	F	G	F	F	F	MP	G
	DM4	G	MG	G	F	F	G	MG	P	F	F	F	MP	G
	DM5	G	MG	G	F	F	VG	F	P	F	F	F	MG	G
	DM6	G	MG	G	F	F	VG	P	MG	F	F	F	G	G
	DM7	G	MG	G	F	F	VG	F	P	F	F	F	MP	G
	DM1	F	VG	F	P	VG	G	F	VG	MG	F	G	F	VG
A2	**DM2**	G	G	VG	F	VG	G	F	VG	G	F	G	F	VG
	DM3	G	G	VG	P	VG	G	F	VG	MG	F	G	F	VG
	DM4	G	G	VG	P	VG	G	F	VG	MG	F	G	F	VG
	DM5	VG	G	VG	P	VG	G	F	VG	MG	F	G	F	VG
	DM6	VG	G	VG	P	VG	G	F	VG	MG	F	G	F	VG
	DM7	VG	G	VG	P	VG	G	F	VG	MG	F	G	F	VG
	DM1	G	F	F	G	MP	G	MG	F	F	P	MG	G	F
A3	**DM2**	VG	G	F	G	MP	VG	MG	F	MG	P	F	G	F
	DM3	VG	G	F	G	MP	VG	MG	F	F	VG	F	G	F
	DM4	VG	G	F	G	MP	VG	MG	F	F	P	F	G	F
	DM5	VG	G	F	G	MP	VG	MG	F	G	P	F	G	F
	DM6	VG	G	F	G	MP	VG	MG	F	F	MG	F	G	F
	DM7	VG	G	F	G	MP	VG	MG	F	G	P	F	G	F
	DM1	F	MG	F	F	F	F	F	F	G	VG	F	F	MG
A4	**DM2**	F	F	F	F	F	F	F	F	F	VG	F	F	P
	DM3	F	F	F	F	F	F	MP	F	G	VG	F	F	G
	DM4	F	F	F	F	F	F	F	P	G	VG	F	F	P
	DM5	F	F	F	F	F	F	F	F	G	VG	F	F	P
	DM6	F	F	F	F	F	F	F	MG	G	VG	F	F	G
	DM7	F	F	F	F	F	F	F	F	G	VG	F	F	P
	DM1	VG	G	G	MG	P	G	VG	G	F	F	MP	P	G
A5	**DM2**	G	MG	G	MG	P	F	G	G	F	F	MP	F	F
	DM3	G	MG	G	MG	P	G	G	MG	F	F	G	P	F
	DM4	VG	MG	G	MG	P	G	G	G	F	F	MP	P	MG
	DM5	VG	MG	G	MG	P	G	G	G	F	F	MG	P	F
	DM6	VG	MG	G	MG	P	G	G	G	F	F	MP	P	G
	DM7	VG	MG	G	MG	P	G	G	G	F	F	MP	P	F
	DM1	G	VG	VG	F	F	G	F	F	F	F	G	F	MP
A6	**DM2**	VG	VG	G	F	F	G	F	F	F	F	G	F	MP
	DM3	VG	VG	G	F	F	G	F	F	F	F	G	F	G
	DM4	VG	G	G	F	F	G	F	F	F	F	G	F	MP
	DM5	G	VG	G	F	F	G	F	F	F	F	G	F	MP
	DM6	G	VG	G	F	F	G	F	F	F	F	G	F	MG
	DM7	G	VG	G	F	F	G	F	F	F	F	G	F	MP

Table 9
Linguistic evaluation terms and IT2Fs for alternatives [9, 10, 18]

Linguistic Terms	Abb.	a_1^u	a_2^u	a_3^u	a_4^u	$h_1^u(A^U)$	$h_2^u(A^U)$	a_1^l	a_2^l	a_3^l	a_4^l	$h_1^l(A^L)$	$h_2^l(A^L)$
Very low	VL	0	0	0	0.1	1	1	0	0	0	0.05	0.9	0.9
Low	L	0	0.1	0.1	0.3	1	1	0.05	0.1	0.1	0.2	0.9	0.9
Medium low	ML	0.1	0.3	0.3	0.5	1	1	0.2	0.3	0.3	0.4	0.9	0.9
Medium	M	0.3	0.5	0.5	0.7	1	1	0.4	0.5	0.5	0.6	0.9	0.9
Medium high	MH	0.5	0.7	0.7	0.9	1	1	0.6	0.7	0.7	0.8	0.9	0.9
High	H	0.7	0.9	0.9	1	1	1	0.8	0.9	0.9	0.95	0.9	0.9
Very High	VH	0.9	1	1	1	1	1	0.95	1	1	1	0.9	0.9

Subsequently, the initial IT2F decision matrices were aggregated using Equation (6), and the resulting IT2F decision matrix is presented in Table 10.

Table 10
IT2F aggregated group decision matrix

	A1	A2
C1	[5.00 8.71 8.71 9.57 1.00 1.00 7.86 8.71 8.71 9.14 0.90 0.90]	[5.29 8.86 8.86 9.57 1.00 1.00 8.07 8.86 8.86 9.21 0.90 0.90]
C2	[3.29 7.29 7.29 9.14 1.00 1.00 6.29 7.29 7.29 8.21 0.90 0.90]	[5.29 9.14 9.14 10.00 1.00 1.00 8.21 9.14 9.14 9.57 0.90 0.90]
C3	[4.43 8.43 8.43 9.57 1.00 1.00 7.43 8.43 8.43 9.00 0.90 0.90]	[6.14 9.29 9.29 9.57 1.00 1.00 8.71 9.29 9.29 9.43 0.90 0.90]
C4	[1.57 5.57 5.57 7.43 1.00 1.00 4.57 5.57 5.57 6.50 0.90 0.90]	[0.14 1.57 1.57 3.57 1.00 1.00 1.00 1.57 1.57 2.57 0.90 0.90]
C5	[1.57 5.57 5.57 7.43 1.00 1.00 4.57 5.57 5.57 6.50 0.90 0.90]	[7.00 10.00 10.00 10.00 1.00 1.00 9.50 10.00 10.00 10.00 0.90 0.90]
C6	[5.86 9.14 9.14 9.57 1.00 1.00 8.50 9.14 9.14 9.36 0.90 0.90]	[5.00 9.00 9.00 10.00 1.00 1.00 8.00 9.00 9.00 9.50 0.90 0.90]
C7	[1.71 5.29 5.29 7.14 1.00 1.00 4.36 5.29 5.29 6.21 0.90 0.90]	[1.00 5.00 5.00 7.00 1.00 1.00 4.00 5.00 5.00 6.00 0.90 0.90]
C8	[1.14 3.00 3.00 4.86 1.00 1.00 2.36 3.00 3.00 3.93 0.90 0.90]	[7.00 10.00 10.00 10.00 1.00 1.00 9.50 10.00 10.00 10.00 0.90 0.90]
C9	[1.00 5.00 5.00 7.00 1.00 1.00 4.00 5.00 5.00 6.00 0.90 0.90]	[3.29 7.29 7.29 9.14 1.00 1.00 6.29 7.29 7.29 8.21 0.90 0.90]
C10	[1.00 5.00 5.00 7.00 1.00 1.00 4.00 5.00 5.00 6.00 0.90 0.90]	[1.00 5.00 5.00 7.00 1.00 1.00 4.00 5.00 5.00 6.00 0.90 0.90]
C11	[1.00 5.00 5.00 7.00 1.00 1.00 4.00 5.00 5.00 6.00 0.90 0.90]	[5.00 9.00 9.00 10.00 1.00 1.00 8.00 9.00 9.00 9.50 0.90 0.90]
C12	[1.14 4.14 4.14 6.00 1.00 1.00 3.21 4.14 4.14 5.07 0.90 0.90]	[1.00 5.00 5.00 7.00 1.00 1.00 4.00 5.00 5.00 6.00 0.90 0.90]
C13	[4.43 8.43 8.43 9.57 1.00 1.00 7.43 8.43 8.43 9.00 0.90 0.90]	[7.00 10.00 10.00 10.00 1.00 1.00 9.50 10.00 10.00 10.00 0.90 0.90]
	A3	**A4**
C1	[6.71 9.86 9.86 10.00 1.00 1.00 9.29 9.86 9.86 9.93 0.90 0.90]	[1.00 5.00 5.00 7.00 1.00 1.00 4.00 5.00 5.00 6.00 0.90 0.90]
C2	[4.43 8.43 8.43 9.57 1.00 1.00 7.43 8.43 8.43 9.00 0.90 0.90]	[1.29 5.29 5.29 7.29 1.00 1.00 4.29 5.29 5.29 6.29 0.90 0.90]
C3	[1.00 5.00 5.00 7.00 1.00 1.00 4.00 5.00 5.00 6.00 0.90 0.90]	[1.00 5.00 5.00 7.00 1.00 1.00 4.00 5.00 5.00 6.00 0.90 0.90]
C4	[5.00 9.00 9.00 10.00 1.00 1.00 8.00 9.00 9.00 9.50 0.90 0.90]	[1.00 5.00 5.00 7.00 1.00 1.00 4.00 5.00 5.00 6.00 0.90 0.90]
C5	[0.00 3.00 3.00 5.00 1.00 1.00 2.00 3.00 3.00 4.00 0.90 0.90]	[1.00 5.00 5.00 7.00 1.00 1.00 4.00 5.00 5.00 6.00 0.90 0.90]
C6	[6.71 9.86 9.86 10.00 1.00 1.00 9.29 9.86 9.86 9.93 0.90 0.90]	[1.00 5.00 5.00 7.00 1.00 1.00 4.00 5.00 5.00 6.00 0.90 0.90]
C7	[3.00 7.00 7.00 9.00 1.00 1.00 6.00 7.00 7.00 8.00 0.90 0.90]	[0.86 4.71 4.71 6.71 1.00 1.00 3.71 4.71 4.71 5.71 0.90 0.90]
C8	[1.00 5.00 5.00 7.00 1.00 1.00 4.00 5.00 5.00 6.00 0.90 0.90]	[1.14 4.71 4.71 6.71 1.00 1.00 3.79 4.71 4.71 5.71 0.90 0.90]
C9	[2.43 6.43 6.43 8.14 1.00 1.00 5.43 6.43 6.43 7.29 0.90 0.90]	[4.43 8.43 8.43 9.57 1.00 1.00 7.43 8.43 8.43 9.00 0.90 0.90]
C10	[2.00 3.14 3.14 4.86 1.00 1.00 2.57 3.14 3.14 4.00 0.90 0.90]	[7.00 10.00 10.00 10.00 1.00 1.00 9.50 10.00 10.00 10.00 0.90 0.90]
C11	[1.29 5.29 5.29 7.29 1.00 1.00 4.29 5.29 5.29 6.29 0.90 0.90]	[1.00 5.00 5.00 7.00 1.00 1.00 4.00 5.00 5.00 6.00 0.90 0.90]
C12	[5.00 9.00 9.00 10.00 1.00 1.00 8.00 9.00 9.00 9.50 0.90 0.90]	[1.00 5.00 5.00 7.00 1.00 1.00 4.00 5.00 5.00 6.00 0.90 0.90]
C13	[1.00 5.00 5.00 7.00 1.00 1.00 4.00 5.00 5.00 6.00 0.90 0.90]	[1.86 4.14 4.14 5.86 1.00 1.00 3.43 4.14 4.14 5.00 0.90 0.90]
	A5	**A6**
C1	[6.43 9.71 9.71 10.00 1.00 1.00 9.07 9.71 9.71 9.86 0.90 0.90]	[4.14 8.14 8.14 9.57 1.00 1.00 7.14 8.14 8.14 8.86 0.90 0.90]
C2	[3.29 7.29 7.29 9.14 1.00 1.00 6.29 7.29 7.29 8.21 0.90 0.90]	[4.43 8.29 8.29 9.57 1.00 1.00 7.36 8.29 8.29 8.93 0.90 0.90]
C3	[5.00 9.00 9.00 10.00 1.00 1.00 8.00 9.00 9.00 9.50 0.90 0.90]	[4.43 8.29 8.29 9.57 1.00 1.00 7.36 8.29 8.29 8.93 0.90 0.90]
C4	[3.00 7.00 7.00 9.00 1.00 1.00 6.00 7.00 7.00 8.00 0.90 0.90]	[3.57 7.57 7.57 9.14 1.00 1.00 6.57 7.57 7.57 8.36 0.90 0.90]
C5	[0.00 1.00 1.00 3.00 1.00 1.00 0.50 1.00 1.00 2.00 0.90 0.90]	[3.57 7.57 7.57 9.14 1.00 1.00 6.57 7.57 7.57 8.36 0.90 0.90]
C6	[4.43 8.43 8.43 9.57 1.00 1.00 7.43 8.43 8.43 9.00 0.90 0.90]	[4.14 8.14 8.14 9.57 1.00 1.00 7.14 8.14 8.14 8.86 0.90 0.90]
C7	[5.29 9.14 9.14 10.00 1.00 1.00 8.21 9.14 9.14 9.57 0.90 0.90]	[3.57 7.57 7.57 9.14 1.00 1.00 6.57 7.57 7.57 8.36 0.90 0.90]
C8	[4.71 8.71 8.71 9.86 1.00 1.00 7.71 8.71 8.71 9.29 0.90 0.90]	[3.57 7.57 7.57 9.14 1.00 1.00 6.57 7.57 7.57 8.36 0.90 0.90]
C9	[1.00 5.00 5.00 7.00 1.00 1.00 4.00 5.00 5.00 6.00 0.90 0.90]	[3.57 7.57 7.57 9.14 1.00 1.00 6.57 7.57 7.57 8.36 0.90 0.90]
C10	[1.00 5.00 5.00 7.00 1.00 1.00 4.00 5.00 5.00 6.00 0.90 0.90]	[3.57 7.57 7.57 9.14 1.00 1.00 6.57 7.57 7.57 8.36 0.90 0.90]
C11	[1.14 4.43 4.43 6.29 1.00 1.00 3.43 4.43 4.43 5.36 0.90 0.90]	[4.14 8.14 8.14 9.57 1.00 1.00 7.14 8.14 8.14 8.86 0.90 0.90]
C12	[0.14 1.57 1.57 3.57 1.00 1.00 1.00 1.57 1.57 2.57 0.90 0.90]	[3.57 7.57 7.57 9.14 1.00 1.00 6.57 7.57 7.57 8.36 0.90 0.90]
C13	[2.43 6.43 6.43 8.14 1.00 1.00 5.43 6.43 6.43 7.29 0.90 0.90]	[3.43 7.29 7.29 8.86 1.00 1.00 6.29 7.29 7.29 8.07 0.90 0.90]

For example, the average fuzzy value of A1 $\left(\tilde{\tilde{\xi}}_{11}\right)$ is computed concerning the C1 criterion by considering the linguistic evaluations performed by decision-makers after these evaluations were converted to the IT2Fs as follows.

$$\tilde{\tilde{\xi}}_{11}=\left(\begin{array}{l}\left(\frac{(7+1+\ldots+5)}{7},\frac{(10+5+\ldots+9)}{7},\frac{(10+5+\ldots+9)}{7},\frac{(10+7+\ldots+10)}{7};\frac{(1+1+\ldots+1)}{7},\frac{(1+1+\ldots+1)}{7}\right),\\ \left(\frac{(9.5+4+\ldots+8)}{7},\frac{(10+5+\ldots+9)}{7},\frac{(10+5+\ldots+9)}{7},\frac{(10+6+\ldots+9)}{7};\frac{(0.9+0.9+\ldots+0.9)}{7},\frac{(0.9+0.9+\ldots+0.9)}{7}\right)\end{array}\right)$$

$$\tilde{\tilde{\xi}}_{11}=\left[(5.00,8.71,8.71,9.57,1,1),(7.86,8.71,8.71,9.14,0.9,0.9)\right]$$

The average fuzzy values for the remaining alternatives were calculated similarly, constructing the fuzzy average group decision matrix. Following this, the matrix elements were normalized by considering the nature of each criterion—whether it represents a cost or a benefit. In this study, C1, C2, C3, C5, C6, and C7 are classified as benefit criteria, while the rest are cost criteria. The resulting IT2F normalized average decision matrix is presented in Table 11.

Table 11
IT2F Normalized average group decision matrix

	A1	**A2**
C1	[0.50 0.87 0.87 0.96 1.00 1.00 0.79 0.87 0.87 0.91 0.90 0.90]	[0.53 0.89 0.89 0.96 1.00 1.00 0.81 0.89 0.89 0.92 0.90 0.90
C2	[0.33 0.73 0.73 0.91 1.00 1.00 0.63 0.73 0.73 0.82 0.90 0.90]	[0.53 0.91 0.91 1.00 1.00 1.00 0.82 0.91 0.91 0.96 0.90 0.90
C3	[0.44 0.84 0.84 0.96 1.00 1.00 0.74 0.84 0.84 0.90 0.90 0.90]	[0.61 0.93 0.93 0.96 1.00 1.00 0.87 0.93 0.93 0.94 0.90 0.90
C4	[0.02 0.03 0.03 0.09 1.00 1.00 0.02 0.03 0.03 0.03 0.90 0.90]	[0.04 0.09 0.09 1.00 1.00 1.00 0.06 0.09 0.09 0.14 0.90 0.90
C5	[0.16 0.56 0.56 0.74 1.00 1.00 0.46 0.56 0.56 0.65 0.90 0.90]	[0.70 1.00 1.00 1.00 1.00 1.00 0.95 1.00 1.00 1.00 0.90 0.90
C6	[0.59 0.91 0.91 0.96 1.00 1.00 0.85 0.91 0.91 0.94 0.90 0.90]	[0.50 0.90 0.90 1.00 1.00 1.00 0.80 0.90 0.90 0.95 0.90 0.90
C7	[0.17 0.53 0.53 0.71 1.00 1.00 0.44 0.53 0.53 0.62 0.90 0.90]	[0.10 0.50 0.50 0.70 1.00 1.00 0.40 0.50 0.50 0.60 0.90 0.90
C8	[0.21 0.33 0.33 0.88 1.00 1.00 0.25 0.33 0.33 0.42 0.90 0.90]	[0.10 0.10 0.10 0.14 1.00 1.00 0.10 0.10 0.10 0.11 0.90 0.90
C9	[0.14 0.20 0.20 1.00 1.00 1.00 0.17 0.20 0.20 0.25 0.90 0.90]	[0.11 0.14 0.14 0.30 1.00 1.00 0.12 0.14 0.14 0.16 0.90 0.90
C10	[0.14 0.20 0.20 1.00 1.00 1.00 0.17 0.20 0.20 0.25 0.90 0.90]	[0.14 0.20 0.20 1.00 1.00 1.00 0.17 0.20 0.20 0.25 0.90 0.90
C11	[0.14 0.20 0.20 1.00 1.00 1.00 0.17 0.20 0.20 0.25 0.90 0.90]	[0.10 0.11 0.11 0.20 1.00 1.00 0.11 0.11 0.11 0.13 0.90 0.90
C12	[0.02 0.03 0.03 0.13 1.00 1.00 0.03 0.03 0.03 0.04 0.90 0.90]	[0.02 0.03 0.03 0.14 1.00 1.00 0.02 0.03 0.03 0.04 0.90 0.90
C13	[0.10 0.12 0.12 0.23 1.00 1.00 0.11 0.12 0.12 0.13 0.90 0.90]	[0.10 0.10 0.10 0.14 1.00 1.00 0.10 0.10 0.10 0.11 0.90 0.90
	A3	**A4**
C1	[0.67 0.99 0.99 1.00 1.00 1.00 0.93 0.99 0.99 0.99 0.90 0.90]	[0.10 0.50 0.50 0.70 1.00 1.00 0.40 0.50 0.50 0.60 0.90 0.90]
C2	[0.44 0.84 0.84 0.96 1.00 1.00 0.74 0.84 0.84 0.90 0.90 0.90]	[0.13 0.53 0.53 0.73 1.00 1.00 0.43 0.53 0.53 0.63 0.90 0.90]
C3	[0.10 0.50 0.50 0.70 1.00 1.00 0.40 0.50 0.50 0.60 0.90 0.90]	[0.10 0.50 0.50 0.70 1.00 1.00 0.40 0.50 0.50 0.60 0.90 0.90]
C4	[0.01 0.02 0.02 0.03 1.00 1.00 0.02 0.02 0.02 0.02 0.90 0.90]	[0.02 0.03 0.03 0.14 1.00 1.00 0.02 0.03 0.03 0.04 0.90 0.90]
C5	[0.00 0.30 0.30 0.50 1.00 1.00 0.20 0.30 0.30 0.40 0.90 0.90]	[0.10 0.50 0.50 0.70 1.00 1.00 0.40 0.50 0.50 0.60 0.90 0.90]
C6	[0.67 0.99 0.99 1.00 1.00 1.00 0.93 0.99 0.99 0.99 0.90 0.90]	[0.10 0.50 0.50 0.70 1.00 1.00 0.40 0.50 0.50 0.60 0.90 0.90]
C7	[0.30 0.70 0.70 0.90 1.00 1.00 0.60 0.70 0.70 0.80 0.90 0.90]	[0.09 0.47 0.47 0.67 1.00 1.00 0.37 0.47 0.47 0.57 0.90 0.90]
C8	[0.14 0.20 0.20 1.00 1.00 1.00 0.17 0.20 0.20 0.25 0.90 0.90]	[0.15 0.21 0.21 0.88 1.00 1.00 0.18 0.21 0.21 0.26 0.90 0.90]
C9	[0.12 0.16 0.16 0.41 1.00 1.00 0.14 0.16 0.16 0.18 0.90 0.90]	[0.10 0.12 0.12 0.23 1.00 1.00 0.11 0.12 0.12 0.13 0.90 0.90]
C10	[0.21 0.32 0.32 0.50 1.00 1.00 0.25 0.32 0.32 0.39 0.90 0.90]	[0.10 0.10 0.10 0.14 1.00 1.00 0.10 0.10 0.10 0.11 0.90 0.90]
C11	[0.14 0.19 0.19 0.78 1.00 1.00 0.16 0.19 0.19 0.23 0.90 0.90]	[0.14 0.20 0.20 1.00 1.00 1.00 0.17 0.20 0.20 0.25 0.90 0.90]
C12	[0.01 0.02 0.02 0.03 1.00 1.00 0.02 0.02 0.02 0.02 0.90 0.90]	[0.02 0.03 0.03 0.14 1.00 1.00 0.02 0.03 0.03 0.04 0.90 0.90]
C13	[0.14 0.20 0.20 1.00 1.00 1.00 0.17 0.20 0.20 0.25 0.90 0.90]	[0.17 0.24 0.24 0.54 1.00 1.00 0.20 0.24 0.24 0.29 0.90 0.90]
	A5	**A6**
C1	[0.64 0.97 0.97 1.00 1.00 1.00 0.91 0.97 0.97 0.99 0.90 0.90]	[0.41 0.81 0.81 0.96 1.00 1.00 0.71 0.81 0.81 0.89 0.90 0.90]
C2	[0.33 0.73 0.73 0.91 1.00 1.00 0.63 0.73 0.73 0.82 0.90 0.90]	[0.44 0.83 0.83 0.96 1.00 1.00 0.74 0.83 0.83 0.89 0.90 0.90]
C3	[0.50 0.90 0.90 1.00 1.00 1.00 0.80 0.90 0.90 0.95 0.90 0.90]	[0.44 0.83 0.83 0.96 1.00 1.00 0.74 0.83 0.83 0.89 0.90 0.90]
C4	[0.02 0.02 0.02 0.05 1.00 1.00 0.02 0.02 0.02 0.02 0.90 0.90]	[0.02 0.02 0.02 0.04 1.00 1.00 0.02 0.02 0.02 0.02 0.90 0.90]
C5	[0.00 0.10 0.10 0.30 1.00 1.00 0.05 0.10 0.10 0.20 0.90 0.90]	[0.36 0.76 0.76 0.91 1.00 1.00 0.66 0.76 0.76 0.84 0.90 0.90]
C6	[0.44 0.84 0.84 0.96 1.00 1.00 0.74 0.84 0.84 0.90 0.90 0.90]	[0.41 0.81 0.81 0.96 1.00 1.00 0.71 0.81 0.81 0.89 0.90 0.90]
C7	[0.53 0.91 0.91 1.00 1.00 1.00 0.82 0.91 0.91 0.96 0.90 0.90]	[0.36 0.76 0.76 0.91 1.00 1.00 0.66 0.76 0.76 0.84 0.90 0.90]
C8	[0.10 0.11 0.11 0.21 1.00 1.00 0.11 0.11 0.11 0.13 0.90 0.90]	[0.11 0.13 0.13 0.28 1.00 1.00 0.12 0.13 0.13 0.15 0.90 0.90]
C9	[0.14 0.20 0.20 1.00 1.00 1.00 0.17 0.20 0.20 0.25 0.90 0.90]	[0.11 0.13 0.13 0.28 1.00 1.00 0.12 0.13 0.13 0.15 0.90 0.90]
C10	[0.14 0.20 0.20 1.00 1.00 1.00 0.17 0.20 0.20 0.25 0.90 0.90]	[0.11 0.13 0.13 0.28 1.00 1.00 0.12 0.13 0.13 0.15 0.90 0.90]
C11	[0.16 0.23 0.23 0.88 1.00 1.00 0.19 0.23 0.23 0.29 0.90 0.90]	[0.10 0.12 0.12 0.24 1.00 1.00 0.11 0.12 0.12 0.14 0.90 0.90]
C12	[0.04 0.09 0.09 1.00 1.00 1.00 0.06 0.09 0.09 0.14 0.90 0.90]	[0.02 0.02 0.02 0.04 1.00 1.00 0.02 0.02 0.02 0.02 0.90 0.90]
C13	[0.12 0.16 0.16 0.41 1.00 1.00 0.14 0.16 0.16 0.18 0.90 0.90]	[0.11 0.14 0.14 0.29 1.00 1.00 0.12 0.14 0.14 0.16 0.90 0.90]

For example, the normalized value of the A1 alternative concerning the C1 criterion is computed as follows.

$$\tilde{\eta}_{11} = \left(\left(\frac{5.00}{10}, \frac{8.71}{10}, \frac{8.14}{10} 71, \frac{9.57}{10}; 1,1 \right), \left(\frac{7.86}{10}, \frac{8.71}{10}, \frac{8.71}{10}, \frac{9.14}{10}; 0.9, 0.9 \right) \right)$$

$$= \left[\left(0.50, 0.87, 0.87, 0.96, 1, 1\right), \left(0.79, 0.87, 0.87, 0.91, 0.9, 0.9\right) \right]$$

Next, the IT2F weighted normalized average group decision matrix was generated using Equation (11), and the resulting matrix is presented in the following table.

Table 12
IT2F weighed normalized average group decision matrix

	A1	A2
C1	[0.27 0.61 0.61 0.78 1.00 1.00 0.49 0.61 0.61 0.69 0.90 0.90]	[0.29 0.62 0.62 0.78 1.00 1.00 0.50 0.62 0.62 0.70 0.90 0.90]
C2	[0.15 0.48 0.48 0.73 1.00 1.00 0.35 0.48 0.48 0.60 0.90 0.90]	[0.25 0.60 0.60 0.80 1.00 1.00 0.46 0.60 0.60 0.70 0.90 0.90]
C3	[0.19 0.52 0.52 0.75 1.00 1.00 0.39 0.52 0.52 0.63 0.90 0.90]	[0.26 0.57 0.57 0.75 1.00 1.00 0.45 0.57 0.57 0.66 0.90 0.90]
C4	[0.01 0.02 0.02 0.07 1.00 1.00 0.01 0.02 0.02 0.02 0.90 0.90]	[0.02 0.06 0.06 0.77 1.00 1.00 0.03 0.06 0.06 0.10 0.90 0.90]
C5	[0.05 0.29 0.29 0.50 1.00 1.00 0.20 0.29 0.29 0.39 0.90 0.90]	[0.24 0.51 0.51 0.67 1.00 1.00 0.41 0.51 0.51 0.59 0.90 0.90]
C6	[0.29 0.61 0.61 0.77 1.00 1.00 0.50 0.61 0.61 0.69 0.90 0.90]	[0.25 0.60 0.60 0.80 1.00 1.00 0.47 0.60 0.60 0.70 0.90 0.90]
C7	[0.08 0.35 0.35 0.58 1.00 1.00 0.25 0.35 0.35 0.46 0.90 0.90]	[0.05 0.33 0.33 0.57 1.00 1.00 0.23 0.33 0.33 0.44 0.90 0.90]
C8	[0.08 0.18 0.18 0.60 1.00 1.00 0.12 0.18 0.18 0.26 0.90 0.90]	[0.04 0.05 0.05 0.10 1.00 1.00 0.05 0.05 0.05 0.06 0.90 0.90]
C9	[0.06 0.11 0.11 0.73 1.00 1.00 0.08 0.11 0.11 0.16 0.90 0.90]	[0.04 0.08 0.08 0.22 1.00 1.00 0.06 0.08 0.08 0.10 0.90 0.90]
C10	[0.04 0.09 0.09 0.66 1.00 1.00 0.06 0.09 0.09 0.14 0.90 0.90]	[0.04 0.09 0.09 0.66 1.00 1.00 0.06 0.09 0.09 0.14 0.90 0.90]
C11	[0.06 0.12 0.12 0.77 1.00 1.00 0.08 0.12 0.12 0.17 0.90 0.90]	[0.04 0.07 0.07 0.15 1.00 1.00 0.05 0.07 0.07 0.08 0.90 0.90]
C12	[0.01 0.01 0.01 0.06 1.00 1.00 0.01 0.01 0.01 0.02 0.90 0.90]	[0.00 0.01 0.01 0.07 1.00 1.00 0.01 0.01 0.01 0.02 0.90 0.90]
C13	[0.04 0.06 0.06 0.15 1.00 1.00 0.05 0.06 0.06 0.08 0.90 0.90]	[0.03 0.05 0.05 0.09 1.00 1.00 0.04 0.05 0.05 0.06 0.90 0.90]
	A3	**A4**
C1	[0.36 0.69 0.69 0.81 1.00 1.00 0.58 0.69 0.69 0.75 0.90 0.90]	[0.05 0.35 0.35 0.57 1.00 1.00 0.25 0.35 0.35 0.45 0.90 0.90]
C2	[0.21 0.55 0.55 0.77 1.00 1.00 0.42 0.55 0.55 0.66 0.90 0.90]	[0.06 0.35 0.35 0.58 1.00 1.00 0.24 0.35 0.35 0.46 0.90 0.90]
C3	[0.04 0.31 0.31 0.55 1.00 1.00 0.21 0.31 0.31 0.42 0.90 0.90]	[0.04 0.31 0.31 0.55 1.00 1.00 0.21 0.31 0.31 0.42 0.90 0.90]
C4	[0.01 0.01 0.01 0.02 1.00 1.00 0.01 0.01 0.01 0.01 0.90 0.90]	[0.01 0.02 0.02 0.11 1.00 1.00 0.01 0.02 0.02 0.03 0.90 0.90]
C5	[0.00 0.15 0.15 0.34 1.00 1.00 0.09 0.15 0.15 0.24 0.90 0.90]	[0.03 0.26 0.26 0.47 1.00 1.00 0.17 0.26 0.26 0.36 0.90 0.90]
C6	[0.34 0.66 0.66 0.80 1.00 1.00 0.54 0.66 0.66 0.73 0.90 0.90]	[0.05 0.34 0.34 0.56 1.00 1.00 0.23 0.34 0.34 0.44 0.90 0.90]
C7	[0.15 0.46 0.46 0.73 1.00 1.00 0.34 0.46 0.46 0.59 0.90 0.90]	[0.04 0.31 0.31 0.55 1.00 1.00 0.21 0.31 0.31 0.42 0.90 0.90]
C8	[0.06 0.11 0.11 0.69 1.00 1.00 0.08 0.11 0.11 0.15 0.90 0.90]	[0.06 0.12 0.12 0.60 1.00 1.00 0.08 0.12 0.12 0.16 0.90 0.90]
C9	[0.05 0.09 0.09 0.30 1.00 1.00 0.07 0.09 0.09 0.12 0.90 0.90]	[0.04 0.07 0.07 0.16 1.00 1.00 0.05 0.07 0.07 0.09 0.90 0.90]
C10	[0.06 0.15 0.15 0.33 1.00 1.00 0.09 0.15 0.15 0.22 0.90 0.90]	[0.03 0.05 0.05 0.09 1.00 1.00 0.04 0.05 0.05 0.06 0.90 0.90]
C11	[0.05 0.11 0.11 0.60 1.00 1.00 0.08 0.11 0.11 0.16 0.90 0.90]	[0.06 0.12 0.12 0.77 1.00 1.00 0.08 0.12 0.12 0.17 0.90 0.90]
C12	[0.00 0.01 0.01 0.01 1.00 1.00 0.00 0.01 0.01 0.01 0.90 0.90]	[0.00 0.01 0.01 0.07 1.00 1.00 0.01 0.01 0.01 0.02 0.90 0.90]
C13	[0.05 0.10 0.10 0.64 1.00 1.00 0.07 0.10 0.10 0.14 0.90 0.90]	[0.06 0.12 0.12 0.35 1.00 1.00 0.08 0.12 0.12 0.16 0.90 0.90]
	A5	**A6**
C1	[0.35 0.68 0.68 0.81 1.00 1.00 0.56 0.68 0.68 0.75 0.90 0.90]	[0.22 0.57 0.57 0.78 1.00 1.00 0.44 0.57 0.57 0.67 0.90 0.90]
C2	[0.15 0.48 0.48 0.73 1.00 1.00 0.35 0.48 0.48 0.60 0.90 0.90]	[0.21 0.54 0.54 0.77 1.00 1.00 0.42 0.54 0.54 0.65 0.90 0.90]
C3	[0.21 0.55 0.55 0.79 1.00 1.00 0.42 0.55 0.55 0.67 0.90 0.90]	[0.19 0.51 0.51 0.75 1.00 1.00 0.38 0.51 0.51 0.63 0.90 0.90]
C4	[0.01 0.01 0.01 0.04 1.00 1.00 0.01 0.01 0.01 0.02 0.90 0.90]	[0.01 0.01 0.01 0.03 1.00 1.00 0.01 0.01 0.01 0.02 0.90 0.90]
C5	[0.00 0.05 0.05 0.20 1.00 1.00 0.02 0.05 0.05 0.12 0.90 0.90]	[0.12 0.39 0.39 0.61 1.00 1.00 0.28 0.39 0.39 0.50 0.90 0.90]
C6	[0.22 0.57 0.57 0.77 1.00 1.00 0.44 0.57 0.57 0.66 0.90 0.90]	[0.21 0.55 0.55 0.77 1.00 1.00 0.42 0.55 0.55 0.65 0.90 0.90]
C7	[0.26 0.60 0.60 0.81 1.00 1.00 0.47 0.60 0.60 0.70 0.90 0.90]	[0.17 0.50 0.50 0.74 1.00 1.00 0.38 0.50 0.50 0.61 0.90 0.90]
C8	[0.04 0.06 0.06 0.15 1.00 1.00 0.05 0.06 0.06 0.08 0.90 0.90]	[0.04 0.07 0.07 0.19 1.00 1.00 0.06 0.07 0.07 0.09 0.90 0.90]
C9	[0.06 0.11 0.11 0.73 1.00 1.00 0.08 0.11 0.11 0.16 0.90 0.90]	[0.04 0.08 0.08 0.20 1.00 1.00 0.06 0.08 0.08 0.10 0.90 0.90]
C10	[0.04 0.09 0.09 0.66 1.00 1.00 0.06 0.09 0.09 0.14 0.90 0.90]	[0.03 0.06 0.06 0.18 1.00 1.00 0.04 0.06 0.06 0.09 0.90 0.90]
C11	[0.06 0.13 0.13 0.68 1.00 1.00 0.09 0.13 0.13 0.20 0.90 0.90]	[0.04 0.07 0.07 0.19 1.00 1.00 0.05 0.07 0.07 0.10 0.90 0.90]
C12	[0.01 0.03 0.03 0.51 1.00 1.00 0.02 0.03 0.03 0.06 0.90 0.90]	[0.00 0.01 0.01 0.02 1.00 1.00 0.00 0.01 0.01 0.01 0.90 0.90]
C13	[0.04 0.08 0.08 0.26 1.00 1.00 0.06 0.08 0.08 0.10 0.90 0.90]	[0.04 0.07 0.07 0.19 1.00 1.00 0.05 0.07 0.07 0.09 0.90 0.90]

The value δ_{11} given in Table 12 is calculated as follows.

$$\delta_{11}=\begin{bmatrix}\left[(0.50,0.87,0.87,0.96,1,1),(0.79,0.87,0.87,0.91,0.9,0.9)\right]\otimes\\ \left[(0.614,0.814,0.814,0.957,1,1),(0.714,0.814,0.814,0.886,0.9,0.9)\right]\end{bmatrix}$$

$$\delta_{11}=[(0.27,0.61,0.61,0.78,1,1),(0.49,0.61,0.61,0.90,0.9,0.9)]$$

Subsequently, the values of the optimality function were calculated. To achieve this, Equation (12) was applied to the benefit criteria, while Equation (13) was used for the cost criteria. The resulting optimality function values are presented in Table 13.

Table 13
The values of the IT2F optimality function for decision alternatives

Benefit criteria/ Optimality functions (O_i^*)

	AU						AL					
	P_1^u	P_2^u	P_3^u	P_4^u	$h_1^u(P^U)$	$h_2^u(P^U)$	P_1^u	P_2^u	P_3^u	P_4^u	$h_1^u(P^U)$	$h_2^u(P^U)$
A1	[(1.046	2.854	2.854	4.109	1	1)]	[(2.173	2.854	2.854	3.452	0.9	0.9)]
A2	[(1.338	3.238	3.238	4.373	1	1)]	[(2.524	3.238	3.238	3.788	0.9	0.9)]
A3	[(1.098	2.827	2.827	3.999	1	1)]	[(2.177	2.827	2.827	3.384	0.9	0.9)]
A4	[(0.284	1.907	1.907	3.280	1	1)]	[(1.317	1.907	1.907	2.550	0.9	0.9)]
A5	[(1.196	2.930	2.930	4.113	1	1)]	[(2.261	2.930	2.930	3.495	0.9	0.9)]
A6	[(1.127	3.057	3.057	4.421	1	1)]	[(2.318	3.057	3.057	3.708	0.9	0.9)]

Cost criteria/ Optimality functions (O_i^-)

	AU						AL					
	P_1^u	P_2^u	P_3^u	P_4^u	$h_1^u(P^U)$	$h_2^u(P^U)$	P_1^u	P_2^u	P_3^u	P_4^u	$h_1^u(P^U)$	$h_2^u(P^U)$
A1	[(0.280	0.592	0.592	3.037	1	1)]	[(0.408	0.592	0.592	0.851	0.9	0.9)]
A2	[(0.217	0.408	0.408	2.068	1	1)]	[(0.297	0.408	0.408	0.569	0.9	0.9)]
A3	[(0.272	0.571	0.571	2.594	1	1)]	[(0.396	0.571	0.571	0.812	0.9	0.9)]
A4	[(0.254	0.492	0.492	2.160	1	1)]	[(0.356	0.492	0.492	0.684	0.9	0.9)]
A5	[(0.254	0.523	0.523	3.022	1	1)]	[(0.366	0.523	0.523	0.763	0.9	0.9)]
A6	[(0.205	0.366	0.366	1.005	1	1)]	[(0.278	0.366	0.366	0.488	0.9	0.9)]

Next, the utility degrees of the alternatives were calculated using Equations (14) and (15), considering the nature of each criterion. The results of this calculation are presented in Table 14.

Table 14
IT2F utility degrees of the benefit and cost criteria of the alternatives

Benefit Criteria /Utility Degree (S_i^*)

	AU						AL					
	P_1^u	P_2^u	P_3^u	P_4^u	$h_1^u(P^U)$	$h_2^u(P^U)$	P_1^u	P_2^u	P_3^u	P_4^u	$h_1^u(P^U)$	$h_2^u(P^U)$
A1	0.237	0.646	0.646	0.929	1	1	0.492	0.646	0.646	0.781	0.9	0.9
A2	0.303	0.732	0.732	0.989	1	1	0.571	0.732	0.732	0.857	0.9	0.9
A3	0.248	0.639	0.639	0.904	1	1	0.492	0.639	0.639	0.765	0.9	0.9
A4	0.064	0.431	0.431	0.742	1	1	0.298	0.431	0.431	0.577	0.9	0.9
A5	0.271	0.663	0.663	0.930	1	1	0.512	0.663	0.663	0.790	0.9	0.9
A6	0.255	0.691	0.691	1.000	1	1	0.524	0.691	0.691	0.839	0.9	0.9

Cost Criteria /Utility Degree (S_i^-)

	AU						AL					
	P_1^u	P_2^u	P_3^u	P_4^u	$h_1^u(P^U)$	$h_2^u(P^U)$	P_1^u	P_2^u	P_3^u	P_4^u	$h_1^u(P^U)$	$h_2^u(P^U)$
A1	0.068	0.346	0.346	0.732	1	1	0.241	0.346	0.346	0.503	0.9	0.9
A2	0.099	0.503	0.503	0.947	1	1	0.361	0.503	0.503	0.691	0.9	0.9
A3	0.079	0.359	0.359	0.755	1	1	0.253	0.359	0.359	0.519	0.9	0.9
A4	0.095	0.417	0.417	0.808	1	1	0.300	0.417	0.417	0.577	0.9	0.9
A5	0.068	0.392	0.392	0.806	1	1	0.269	0.392	0.392	0.561	0.9	0.9
A6	0.204	0.560	0.560	1.000	1	1	0.421	0.560	0.560	0.737	0.9	0.9

The defuzzification process was carried out after calculating utility degrees for the alternatives. Subsequently, the final and normalized scores for each alternative were computed. The decision alternatives were ranked based on these relatively significant scores, as presented in Table 15.

Table 15
IT2F - ARAS method conclusions

	Θ_i^*	Θ_i^-	Δ_i	Ω_i	**Rank**
A1	0.962	0.494	0.728	0.745	5
A2	1.127	0.757	0.942	0.964	2
A3	0.948	0.520	0.734	0.751	4
A4	0.576	0.604	0.590	0.604	6
A5	0.999	0.568	0.783	0.802	3
A6	1.062	0.893	0.977	1.000	1

When the results shown in Table 15 are evaluated, it is seen that the most effective and priority strategy is the A6 Digital Transformation and Technology Investment strategy. It is followed by A2 Expanding into New Markets and A5 Transformation of the Organizational Structure, respectively. Other strategies are A3 Strategic Partnerships and Mergers > A1 Expanding the Range of Services > A4 Development of Competitive Strategies.

4. Conclusions

Digital transformation and Industry 4.0 are becoming increasingly important in the independent audit industry. This study developed a model based on IVq-ROF, a multi-criteria decision-making approach, to support the strategic decisions of independent audit firms in the context of digital transformation. First, 13 basic criteria were determined in line with the information obtained from the literature and expert opinions, and then these criteria were weighted using mathematical operations related to IT2F sets. Finally, considering these criteria, six strategic alternatives were evaluated and ranked with the IT2F-ARAS method. The study reveals that independent audit firms should take a multidimensional approach to determining their digital transformation strategies.

When the findings are evaluated, the C1 - Expanding Customer Base criterion is at the top, followed by the C6 - Technological Alignment and C7 - Providing Competitive Advantage criteria, which provide essential insights into the strategic priorities of audit firms in the digital transformation process. Expanding the customer base in the first place shows that digitalization is shaped not only by operational efficiency or regulatory compliance but also by the goal of market access and growth. In this context, it can be emphasized that digital transformation strategies should be structured to optimize internal processes and customer-oriented growth at the managerial level.

The technological adaptation (C6) criterion, which ranks second, reflects the necessity of companies to adapt to rapidly developing technologies. The fact that audit processes are increasingly integrated into artificial intelligence, big data analytics, and digital platforms reveals that this alignment is critical to sustainability. The fact that the competitive advantage (C7) criterion ranks third also supports this tendency. Companies consider technology not only as a means of adaptation but also as a strategic element that will create differentiation in the market. The prominence of these three criteria indicates that firms adopt a proactive strategy, not a reactive one. In other words, it reveals that they see digital transformation not as a necessity but as an opportunity to create a competitive advantage.

The fact that criteria such as Contribution to Service Quality (C2) and Brand and Reputation Increase (C3) are also at the top shows that long-term values such as increasing customer satisfaction and building trust maintain their importance in the sector. It reveals a two-pronged understanding of strategy focusing on tangible deliverables (e.g., quality of service) and intangible assets (e.g., brand reputation).

On the other hand, the fact that criteria such as Technological Infrastructure Incompatibility (C4) and Operational Complexity (C11) are in the middle ranks shows that companies are aware of the internal difficulties they may encounter in the digitalization process. Still, they evaluate them in the second plan. Initial Investment Cost (C9), Sustainability Risk (C5), and Regulatory Compliance Risk (C13), which are ranked lower, show that firms prioritize long-term benefits rather than short-term cost or compliance risks.

Training and Human Resource Requirements (C10) and Implementation Time (C12), which are at the bottom, are seen as more manageable elements by companies, or their importance levels may not be sufficiently understood. However, this is an essential point of attention for managers, as it can lead to serious capacity problems during the implementation phase. These criteria should not be ignored as they can directly affect the highest priority outputs.

This ranking shows that audit firms focus on innovation and market expansion in their strategies for digital transformation. For policymakers, these findings highlight the importance of publicly supported innovation funds, digital infrastructure incentives, and technology programs that can be carried out jointly with the private sector. On the other hand, it would be beneficial for legislators to turn to capacity-building regulations that are not only limited to audit and compliance frameworks but also support customer-oriented digital innovation.

In terms of strategic management, it is recommended that audit firms plan their digital transformation roadmaps in line with their customer base expansion goals and with strong technology integration. Operational challenges of moderate importance can be managed through phased implementation and stakeholder engagement. Criteria such as training and implementation time, which are considered low priority, should be regularly reviewed as they can determine the success of the transformation in the long run.

Regarding strategy alternatives, the A6 Digital Transformation and Technology Investment strategy has been determined as the most prominent and priority strategy. This finding strongly aligns with both the literature and the global orientation of digitalization. Digital transformation in the audit industry is not only a technological transition; It also means structuring new business models, service delivery methods, and data-based decision mechanisms. Therefore, the fact that this strategy is in the first place shows the determination of companies to invest in digital infrastructure to maintain their long-term competitiveness. From a managerial point of view, this situation means that technology investments are not only for operational improvements; They also reveal that they are the cornerstone of strategic growth.

The A2 – Expanding New Markets strategy is the second, directly related to expanding the customer base and increasing revenue diversity. It is consistent with criterion C1 being the highest priority criterion. The digital infrastructure's scalability and remote service opportunity allow audit firms to reach broader audiences geographically. In this context, firms aim not only to increase competition at the local level but also to increase global competitiveness.

The third A5 – Transformation of Organizational Structure strategy reflects the efforts to adapt to the new processes and ways of doing business that come with digitalization. Technological transformation is not limited to software or hardware investment; It also requires organizational flexibility, digital leadership, agile management, and integrating innovative culture into corporate structures. The fact that this strategy is at the top reveals that companies attach importance to establishing an organizational infrastructure compatible with technology.

On the other hand, the fact that the A3 – Strategic Partnerships and Mergers strategy is in the middle ranks shows that companies tend to carry out digital transformation with their internal resources or are cautious about such collaborations. However, such partnerships can offer significant advantages in rapidly adapting to digitalization, especially for small and medium-sized audit firms with limited digital capacity.

A1 – Expansion of Service Range and A4 – Development of Competitive Strategies, which are in the lower ranks, are strategies that have limited impact in the short term or are considered less of a priority in digital transformation. However, expanding the range of services can be essential for evaluating the new service capacity that emerges after technological investments. On the other hand, competitive strategies can be restructured to increase differentiation after the transformation is completed.

This ranking shows that audit firms prioritize developing internal capacity and investing in new technologies in the digital transformation process. From the point of view of policymakers, this situation indicates that companies may need cost and risk sharing to install digital infrastructure. In this context, digitalization programs supported by technology investment incentives, tax breaks, or public-private partnership models can contribute to the sector's development.

In terms of strategic management, the digital transformation process of companies is not only focused on technology; It is recommended that they handle it with a holistic approach, such as human resources, organizational flexibility, and market expansion. This way, internal productivity and external growth targets can be achieved healthily.

This study provides multifaceted contributions in evaluating digital transformation and Industry 4.0 applications in the independent audit sector and integrating them into strategic decision support processes. First, a novel decision model was developed using the IT2F-ARAS method, an advanced version of multi-criteria decision-making (MCDM) approaches considering ambiguity and linguistic evaluations. This methodological approach brings an essential innovation to the literature in terms of modeling the uncertainties in assessments based on expert opinions more realistically.

In addition, this study provides concrete insights to decision-makers by systematically analyzing the priority criteria of digitalization in the independent audit sector. By determining the importance levels of the criteria, it has been revealed which elements play a more critical role in digital transformation strategies. For example, factors such as expanding the customer base, technological adaptation, and providing a competitive advantage are at the forefront, showing that companies are turning to digital strategies focused on growth and competitiveness. This finding offers a valuable roadmap for both managers and policymakers.

The systematic ranking of strategy alternatives directly contributes to strategic planning by revealing which digitalization steps companies should prioritize. In particular, the fact that "Digital transformation and technology investments" have been determined as the top priority strategy shows that digitalization is seen not only as a supporting element but also as a necessity in the sector.

An essential advantage of this study is that it integrates both qualitative (expert assessments) and quantitative (IT2F digitization and optimization) data, resulting in a comprehensive analysis. Thanks to this multi-layered structure, the evaluation criteria and the feasible strategies can be rationally compared. In this respect, the study contributes to the academic literature; It is also a decision support system for practitioners.

Finally, one of the innovative aspects of the study is that it examines the sector-specific strategic transformation issue in the context of the independent audit sector, which has been discussed in a limited way in the literature. In this respect, the study fills the gaps in the literature and lays the groundwork for future studies methodologically and thematically. Although this study presents essential findings by providing a comprehensive analysis, it has some limitations, which impact the validity and generalizability of the findings.

First, the data used in the study is based on expert opinions, and the perceptions and experiences of a limited number of participants shape the evaluations. Although experts are competent in their field, studies with a larger and more diverse group of experts from different countries, company scales (e.g., international audit firms and local small and medium-sized audit offices), or other levels of experience can provide more generalizable results.

Secondly, the research focuses on the independent audit sector. Criterion priorities and strategic recommendations developed by considering these sector-specific dynamics may not directly apply to digital transformation processes in other sectors. For example, the priorities and challenges associated with digitalization can vary considerably in different industries, such as manufacturing, healthcare, or logistics. For this reason, the study's results should be evaluated in a specific context, and adaptations may be required for applications to be made in other sectors.

Third, although the method used in the study, the IT2F-ARAS model, provides a strong analytical framework, such methods operate under some assumptions. Interval-type quadratic fuzzy set structures are used to a certain extent in modeling uncertainties and abstract real-world complexities. In addition, it should be borne in mind that these steps have subjective aspects since the choices made in the assignment of mathematical equivalents of linguistic variables can affect the results. Finally, this study analyzes a dynamic process, i.e., digital transformation, at a static time. However, digitalization is a process that develops and changes over time. Therefore, the importance of criteria and strategies may differ over time. Future longitudinal studies may allow us to monitor this change better and evaluate the strategy's effectiveness over time. Although these limitations do not diminish the study's validity, they require careful interpretation of the findings obtained and appropriate adaptations when transferring them to different contexts.

Author Contributions

"Conceptualization, O.F. Görçün and; methodology, O.F. Görçün; software; validation O.F. Görçün; formal analysis; investigation, C.N. Durmuş; resources, C.N. Durmuş; data curation, C.N. Durmuş; writing—original draft preparation, O.F. Görçün and C.N. Durmuş; writing—review and editing, O.F.Gorcun and C.N. Durmuş; project administration, O.F. Görçün and C.N Durmuş. All authors have read and agreed to the published version of the manuscript."

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Data Availability Statement

The data used in this study were obtained from publicly available financial reports and balance sheets of banks operating in the Balkan countries. Based on these data, decision-makers conducted linguistic evaluations using predefined linguistic scales to assess the performance of the banks across specified criteria. The linguistic evaluation process and the associated data can be made available by contacting the authors upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this study.

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