



# Enhancing Cybersecurity Risk Assessment Techniques Based on Bipolar Complex Intuitionistic Fuzzy Soft Robust Aggregation Operators: A Comprehensive Approaches

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## ABSTRACT

This article describes the cybersecurity risk assessment model's accuracy, timely response, and actions against potential threats by providing the key areas in which the cybersecurity risk assessment model can be improved and enhanced. For this, we propose the model of robust aggregation operators based on bipolar complex intuitionistic fuzzy soft information, called the bipolar complex intuitionistic fuzzy soft weighted averaging operator, bipolar complex intuitionistic fuzzy soft order weighted averaging operator, bipolar complex intuitionistic fuzzy soft weighted geometric operators, and bipolar complex intuitionistic fuzzy soft order weighted geometric operators, with some practical properties, called idempotency, monotonicity, and boundedness. Further, we discuss the multi-attribute decision-making model based on proposed operators for evaluating the enhancement of the cybersecurity risk assessment techniques with a comprehensive approach. Finally, we illustrate numerical examples based on decision-making techniques for assessing the comparative analysis among the proposed ranking values with existing ranking values to show the worth and verity of the designed models.

## 1. Introduction

Cybersecurity threats [1] are a great risk nowadays for every organization in this digital fast-paced world. To stay ahead in the business these types of threats whether in the form of data breaches, ransomware or advanced digital threats are making it difficult for organizations. As of these continuously changing threats sophistication and increased frequency organizations need their risk assessment models to become more robust, adaptable, and comprehensive to counter the data breaches. To eliminate modern complexities of cybersecurity threats traditional approaches to cybersecurity risk assessment are no longer useful [2]. There is a great need for organizations to invest improve and enhance their cybersecurity risk assessment models by incorporating new technologies, methodologies, and advanced strategies to effectively eliminate these risks. Input from

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all departments of organizations is important for cybersecurity not just the IT department this trend has to be changed because the IT department is only responsible for the cybersecurity, departmental collaboration is very crucial for the optimal cybersecurity model so that all the potential threats are monitored and eliminated on time [3]. For example, the factors by which a risk profile of an organization can be influenced significantly include human factors such as awareness and behavioral aspects of an employee. Cyber-attacks are more susceptible to weak passwords and weak security practices. Regular awareness sessions amongst employees organized by the HR and training department in the risk assessment model process can significantly reduce these human-based risks. With the ever-changing nature of cyber threats, the adaptability and dynamics of a risk assessment model should keep changing at a fast pace [4]. Emerging threats can quickly make a static risk assessment model vulnerable because it was updated annually and became outdated quickly. For organizations to have a more reliable and dynamic risk assessment model their risk profile should evolve dynamically according to the updated data and emerging cyber threats. The relevance and effectiveness of the organization's risk assessment model can be improved by continuous and real-time monitoring of cyber threats [5]. AI-based automated assessment models that incorporate data in real-time from different sources can be really helpful for organizations to respond to and eliminate new threats quickly as they emerge.

The decision-making model is a very classified and famous structure for the assessment of the best preference among the collection of preferences based on some attributes. Decision-making techniques have contained many reliable and dominant types, for instance, clustering analysis, image segmentation, and multi-attribute decision-making (MADM) model, also used for the same assessments, but with different criteria and features. Many individuals have utilized the technique of the MADM tool based on classical information and due to these aspects, experts have lost a lot of data because of limited and bounded options. From the structure of classical set theory, we observed that they have just two options such as zero and one, but neither. In 1965, Zadeh [6] evaluated the solution to the above problems for the first time in the form of fuzzy set (FS) theory, the FS theory contained a function of truth degree, defined from any universe of discourse to unit interval. Further, we also observed that the truth function or truth degree is not enough to deal with many real-life problems, because the falsity degree or falsity function is also a very important part of genuine-life problems, for this, Atanassov [7, 8] invented the theory of intuitionistic fuzzy set (IFS) theory, the structure of IFS theory is defined with two different functions but with the same range, called the function of truth degree and the function of falsity degree with a condition that is the sum of both functions must be part of unit interval. Xiong et al. [9] designed the conditional aggregation information based on type-2 fuzzy systems. Ali and Hila [10] developed the power operators based on Aczel-Alsina norms for fuzzy 2-tuple linguistic information.

When a person wants to buy any type of software from a software shop, they need two types of data, (i) name of the required software, and (ii) version of the required software. Without this information our question is incomplete, this information can be described by complex numbers because the name is represented by the real part, and the version is represented by the imaginary part. The main question is what happens if we change the range of truth function to a complex number instead of a real number? For this, Ramot et al. [11] designed the truth function in the form of a complex number in the structure of complex fuzzy sets (CFSs), where the real and imaginary parts of the complex-valued truth function are defined from any fixed set to unit interval. But still, there are some problems like IFSs, the complex-valued truth function is not enough to cope with awkward and vague

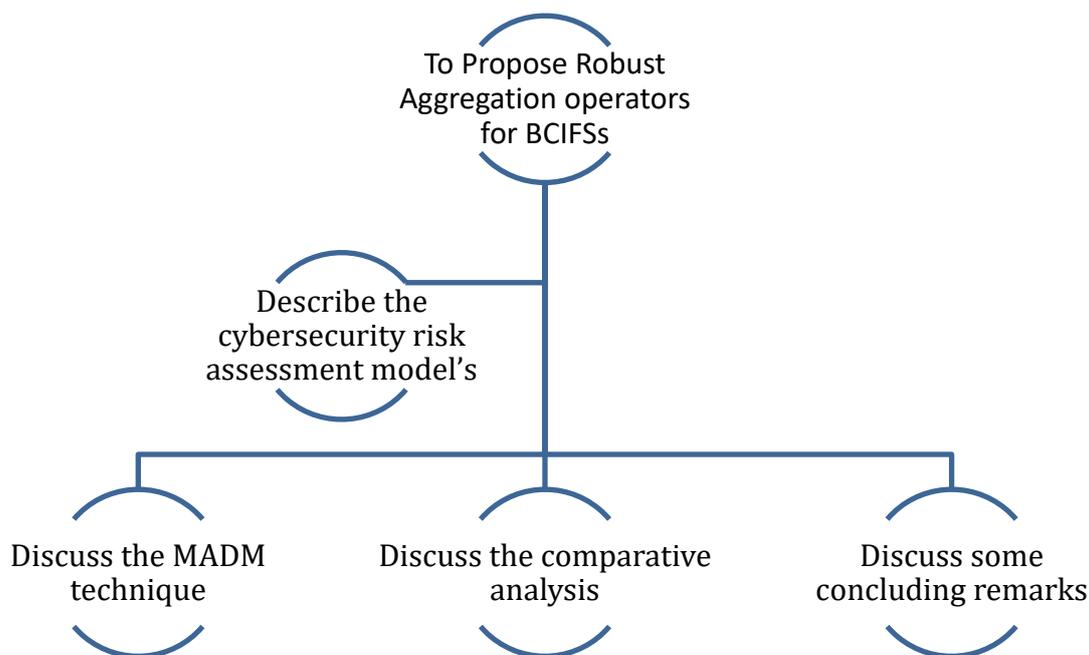
data, for this, Alkouri and Salleh [12] designed the structure of complex intuitionistic fuzzy sets (CIFs). The structure of CIFs is very famous and reliable because they cope with two functions, called the complex-valued truth function and the complex-valued falsity function with a condition that the sum of the real part (imaginary part) of both functions must be part of the unit interval. Mahmood et al. [13] evaluated the complex intuitionistic fuzzy rough set with application. Bisht et al. [14] developed the novel measures for IFs with entropy measures. Ali et al. [15] proposed the Archimedean Heronian mean operators for CIFs.

The technique of fuzzy information is a very prominent and flexible scenario because of the positive truth function, but it is also very clear that every decision, option, and things have positive as well as negative impacts, the fuzzy information has just talked about positive but ignored the negative one. For this, Zhang [16] developed the theory of bipolar fuzzy sets (BFS) with two different functions with two different ranges, called the positive truth function and the negative truth function with a range  $[0, 1]$  and  $[-1, 0]$ . The positive truth function and negative truth functions are not enough to cope with awkward and complex information, for this Mahmood and Rehman [17] proposed the theory of bipolar complex fuzzy sets (BCFs). The technique of BCFs is very famous and dominant because of their features, where the information in BCFs is computed in the form of complex numbers, which can easily deal with two-dimensional information in genuine-life problems.

Lee et al. [18] invented the model of bipolar-valued fuzzy information with interval-valued fuzzy and intuitionistic fuzzy information. Molodtsov [19] developed the theory of soft sets (SSs) in 1999, with a function defined from the set of parameters to the power set of the universal set. Abdullah et al. [20] developed the bipolar fuzzy soft sets by combining the technique of BFSs with SSs to cope with complex problems. Alqaraleh et al. [21] designed the bipolar complex fuzzy soft sets by maxing the technique of BFSs, and CFSs, with SSs for handling complicated and problematic information. Jana and Pal [22] evaluate the bipolar intuitionistic fuzzy soft sets (BIFSS). Ali [23] developed the novel technique of bipolar complex intuitionistic fuzzy soft (BCIFS) sets which is the modified version of all existing techniques, already discussed in the previous literature review. The technique of BCIFS is very dominant and reliable in coping with vague and uncertain data in genuine-life problems. Xu [24] developed the aggregation operators for IFs. Xu and Yager [25] proposed the intuitionistic fuzzy geometric operators. Arora and Garg [26] invented the intuitionistic fuzzy soft robust operators. Jana et al. [27] designed the bipolar fuzzy soft robust operators. Ali and Yang [28] proposed the bipolar complex fuzzy soft robust operators. Gwak et al. [29] explored the hybrid structure based on bipolar complex fuzzy soft sets. Riaz and Tehrim [30] described the cubic bipolar fuzzy geometric operators. The major contribution of this manuscript is described below:

- 1) To describe the cybersecurity risk assessment model's accuracy, timely response, and actions against potential threats by providing the key areas in which the cybersecurity risk assessment model can be improved and enhanced.
- 2) To propose the model of robust aggregation operators based on BCIFS information, called the BCIFS weighted averaging (BCIFSWA) operator, BCIFS order weighted averaging (BCIFSOWA) operator, BCIFS weighted geometric (BCIFSWG) operators, and BCIFS order weighted geometric (BCIFSOWG) operators, with some practical properties, called idempotency, monotonicity, and boundedness.

- 3) To discuss the multi-attribute decision-making (MADM) model based on proposed operators for evaluating the enhancement of the cybersecurity risk assessment techniques with a comprehensive approach.
- 4) To illustrate numerical examples based on decision-making techniques for assessing the comparative analysis among the proposed ranking values with existing ranking values to show the worth and verity of the designed models. **The graphical abstract of the proposed theory is mentioned in Figure 1.**



**Figure 1. Graphical interpretation of the proposed theory.**

This manuscript is arranged in the following form, In Section 2, we revised the information on BCIFSSs with some operational laws. In Section 3, we proposed the model of robust aggregation operators based on BCIFS information, called the BCIFSWA operator, BCIFSOWA operator, BCIFSWG operators, and BCIFSOWG operators, with some practical properties, called idempotency, monotonicity, and boundedness. In Section 4, we discussed the MADM model based on proposed operators for evaluating the enhancement of the cybersecurity risk assessment techniques with a comprehensive approach. In Section 5, we illustrated numerical examples based on decision-making techniques for assessing the comparative analysis among the proposed ranking values with existing ranking values to show the worth and verity of the designed models in Section 6. Some concluding remarks are stated in Section 7.

## 2. Preliminaries

In this section, we revised the information on BCIFSSs with some operational laws.

Definition 1: [23] Consider a fixed set  $\mathbb{Y}$  with a set of parameters  $\mathbb{A}$  such as  $\mathbb{E} \subset \mathbb{A}$ . Thus, the BCIFSS  $\mathbb{A}_{BCIFS}$  is mentioned and explored by:

$$\mathbb{A}_{BCIFS} = \left\{ \left( \ddot{\alpha}_I, \left( \mathbb{C}^R(\ddot{\alpha}I) + L\mathbb{C}^I(\ddot{\alpha}I), \mathbb{D}^R(\ddot{\alpha}I) + L\mathbb{D}^I(\ddot{\alpha}I), \right. \right. \right. \\ \left. \left. \left. \mu^R(\ddot{\alpha}I) + L\mu^I(\ddot{\alpha}I), \eta^R(\ddot{\alpha}I) + L\eta^I(\ddot{\alpha}I) \right) \right) : \ddot{\alpha}_I \in \mathbb{Y}, L = \sqrt{-1} \right\}$$

Thus, the duplet  $(\mathbb{A}_{BCIFS}, \mathbb{E})$  represents a BCIFSS, such as  $\mathbb{A}_{BCIFS\mathbb{Z}}: \mathbb{E} \rightarrow BCIFS(\mathbb{Y})$ . The function in CIFSS is denoted and defined by:  $\mathbb{C}^R, \mathbb{C}^I, \mu^R, \mu^I: \mathbb{Y} \rightarrow [0, 1]$  and  $\mathbb{D}^R, \mathbb{D}^I, \eta^R, \eta^I: \mathbb{Y} \rightarrow [-1, 0]$  with  $\mathbb{C}^R(\ddot{\alpha}), \mathbb{C}^I(\ddot{\alpha}) \in [0, 1]$ ,  $\mu^R(\ddot{\alpha}), \mu^I(\ddot{\alpha}) \in [0, 1]$  and  $\mathbb{D}^R(\ddot{\alpha}), \mathbb{D}^I(\ddot{\alpha}) \in [-1, 0]$ ,  $\eta^R(\ddot{\alpha}), \eta^I(\ddot{\alpha}) \in [-1, 0]$ . Finally, the BCIFSN is mentioned by:  $\mathbb{A}_{BC}^{IJ} = (\mathbb{C}_{IJ}^R + L\mathbb{C}_{IJ}^I, \mathbb{D}_{IJ}^R + L\mathbb{D}_{IJ}^I, \mu_{IJ}^R + L\mu_{IJ}^I, \eta_{IJ}^R + L\eta_{IJ}^I)$ ,  $I, J = 1, 2, \dots, \tilde{\alpha}, \mathfrak{y}$ .

Definition 2: [23] Considering any two BCIFSNs for the evaluation of the algebraic operational laws, such as

$$\mathbb{A}_{BC}^{11} \oplus \mathbb{A}_{BC}^{12} = \left( \begin{array}{l} (\mathbb{C}_{11}^R + \mathbb{C}_{12}^R - \mathbb{C}_{11}^R \mathbb{C}_{12}^R) + L(\mathbb{C}_{11}^I + \mathbb{C}_{12}^I - \mathbb{C}_{11}^I \mathbb{C}_{12}^I), \\ (\mathbb{D}_{11}^R + \mathbb{D}_{12}^R - \mathbb{D}_{11}^R \mathbb{D}_{12}^R) + L(\mathbb{D}_{11}^I + \mathbb{D}_{12}^I - \mathbb{D}_{11}^I \mathbb{D}_{12}^I), \\ \mu_{11}^R \mu_{12}^R + L\mu_{11}^I \mu_{12}^I, -|\eta_{11}^R| |\eta_{12}^R| + L(-|\eta_{11}^I| |\eta_{12}^I|) \end{array} \right)$$

$$\mathbb{A}_{BC}^{11} \otimes \mathbb{A}_{BC}^{12} = \left( \begin{array}{l} \mathbb{C}_{11}^R \mathbb{C}_{12}^R + L\mathbb{C}_{11}^I \mathbb{C}_{12}^I, -|\mathbb{D}_{11}^R| |\mathbb{D}_{12}^R| + L(-|\mathbb{D}_{11}^I| |\mathbb{D}_{12}^I|), \\ (\mu_{11}^R + \mu_{12}^R + \mu_{11}^R \mu_{12}^R) + L(\mu_{11}^I + \mu_{12}^I + \mu_{11}^I \mu_{12}^I), \\ (\eta_{11}^R + \eta_{12}^R + \eta_{11}^R \eta_{12}^R) + L(\eta_{11}^I + \eta_{12}^I + \eta_{11}^I \eta_{12}^I) \end{array} \right)$$

$$\mathbb{Z}_{2C} \mathbb{A}_{BC}^{11} = \left( \begin{array}{l} (1 - (1 - \mathbb{C}_{11}^R)^{2_{2C}}) + L(1 - (1 - \mathbb{C}_{11}^I)^{2_{2C}}), \\ -1 + |1 + \mathbb{D}_{11}^R|^{2_{2C}} + L(-1 + |1 + \mathbb{D}_{11}^I|^{2_{2C}}), \\ (\mu_{11}^R)^{2_{2C}} + L(\mu_{11}^I)^{2_{2C}}, -|\eta_{11}^R|^{2_{2C}} + L(-|\eta_{11}^I|^{2_{2C}}) \end{array} \right)$$

$$(\mathbb{A}_{BC}^{11})^{2_{2C}} = \left( \begin{array}{l} (\mathbb{C}_{11}^R)^{2_{2C}} + L(\mathbb{C}_{11}^I)^{2_{2C}}, -|\mathbb{D}_{11}^R|^{2_{2C}} + L(-|\mathbb{D}_{11}^I|^{2_{2C}}), \\ (1 - (1 - \mu_{11}^R)^{2_{2C}}) + L(1 - (1 - \mu_{11}^I)^{2_{2C}}), \\ -1 + |1 + \eta_{11}^R|^{2_{2C}} + L(-1 + |1 + \eta_{11}^I|^{2_{2C}}) \end{array} \right)$$

Definition 3: [23] Considering any BCIFSN for the evaluation of the Score and Accuracy laws, such as

$$\mathcal{ZC}(\mathbb{A}_{BC}^{IJ}) = \frac{1}{8} \left( (\mathbb{C}_{IJ}^R + \mathbb{C}_{IJ}^I + \mathbb{D}_{IJ}^R + \mathbb{D}_{IJ}^I) - (\mu_{IJ}^R + \mu_{IJ}^I + \eta_{IJ}^R + \eta_{IJ}^I) \right) \in [-1, 1]$$

$$AC(\mathbb{A}_{BC}^{IJ}) = \frac{1}{8} \left( (\mathbb{C}_{IJ}^R + \mathbb{C}_{IJ}^I - \mathbb{D}_{IJ}^R - \mathbb{D}_{IJ}^I) + (\mu_{IJ}^R + \mu_{IJ}^I - \eta_{IJ}^R - \eta_{IJ}^I) \right) \in [0, 1]$$

Where, the following information is also very important for the above theory, such as: When  $\mathcal{ZC}(\mathbb{A}_{BC}^{11}) > \mathcal{ZC}(\mathbb{A}_{BC}^{12})$ , thus  $\mathbb{A}_{BC}^{11} > \mathbb{A}_{BC}^{12}$ , When  $AC(\mathbb{A}_{BC}^{11}) > AC(\mathbb{A}_{BC}^{12})$ , thus  $\mathbb{A}_{BC}^{11} > \mathbb{A}_{BC}^{12}$ .

### 3. BCIFS Robust Aggregation Operators

In this section, we investigate the information of the BCIFSWA operator, BCIFSOWA operator, BCIFSWG operator, and BCIFSOGA operator with their basic laws, called idempotency, monotonicity, and boundedness.

Definition 4: Consider any family of BCIFSNs  $A_{BC}^{11}, A_{BC}^{12}, \dots, A_{BC}^{\tilde{\alpha}^{\alpha}}$ . Thus, the technique of BCIFSWA operator based on  $A_{BC}^{11}, A_{BC}^{12}, \dots, A_{BC}^{\tilde{\alpha}^{\alpha}}$  is mentioned and developed by:

$$BCIFSWA(A_{BC}^{11}, A_{BC}^{12}, \dots, A_{BC}^{\tilde{\alpha}^{\alpha}}) = \bigoplus_{J=1}^{\alpha} z_{\Omega}^J \left( \bigoplus_{I=1}^{\tilde{\alpha}} z_{\psi}^I A_{BC}^{IJ} \right)$$

The mathematical notion of the weight vector is as follows, such as  $z_{\Omega}^J, z_{\psi}^I \in [0, 1]$  with  $\sum_{I=1}^{\tilde{\alpha}} z_{\psi}^I = 1$  and  $\sum_{J=1}^{\alpha} z_{\Omega}^J = 1$ .

Theorem 1: Prove that the aggregated value of the  $BCIFSWA(A_{BC}^{11}, A_{BC}^{12}, \dots, A_{BC}^{\tilde{\alpha}^{\alpha}})$  is also a BCIFSN, such as

$$BCIFSWA(A_{BC}^{11}, A_{BC}^{12}, \dots, A_{BC}^{\tilde{\alpha}^{\alpha}}) = \left( \begin{array}{l} \left( 1 - \prod_{J=1}^{\alpha} \left( \prod_{I=1}^{\tilde{\alpha}} (1 - \mathfrak{C}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( 1 - \prod_{J=1}^{\alpha} \left( \prod_{I=1}^{\tilde{\alpha}} (1 - \mathfrak{C}_{IJ}^I)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( -1 + \prod_{J=1}^{\alpha} \left( \prod_{I=1}^{\tilde{\alpha}} (1 + \mathfrak{D}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( -1 + \prod_{J=1}^{\alpha} \left( \prod_{I=1}^{\tilde{\alpha}} (1 + \mathfrak{D}_{IJ}^I)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( \prod_{J=1}^{\alpha} \left( \prod_{I=1}^{\tilde{\alpha}} (\mathfrak{M}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( \prod_{J=1}^{\alpha} \left( \prod_{I=1}^{\tilde{\alpha}} (\mathfrak{M}_{IJ}^I)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( -\prod_{J=1}^{\alpha} \left( \prod_{I=1}^{\tilde{\alpha}} (|\mathfrak{N}_{IJ}^R|)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( -\prod_{J=1}^{\alpha} \left( \prod_{I=1}^{\tilde{\alpha}} (|\mathfrak{N}_{IJ}^I|)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right). \end{array} \right)$$

Proof: Using mathematical induction prove the above theory. Let  $\tilde{\alpha} = 1$  and  $z_{\psi}^I = 1$ , then

$$BCIFSWA(A_{BC}^{11}, A_{BC}^{12}, \dots, A_{BC}^{1\alpha}) = \bigoplus_{J=1}^{\alpha} z_{\Omega}^J (A_{BC}^{1J}) = \left( \begin{array}{l} \left( 1 - \prod_{J=1}^{\alpha} (1 - \mathfrak{C}_{1J}^R)^{z_{\Omega}^J} \right) + L \left( 1 - \prod_{J=1}^{\alpha} (1 - \mathfrak{C}_{1J}^I)^{z_{\Omega}^J} \right), \\ \left( -1 + \prod_{J=1}^{\alpha} (1 + \mathfrak{D}_{1J}^R)^{z_{\Omega}^J} \right) + L \left( -1 + \prod_{J=1}^{\alpha} (1 + \mathfrak{D}_{1J}^I)^{z_{\Omega}^J} \right), \\ \left( \prod_{J=1}^{\alpha} (\mathfrak{M}_{1J}^R)^{z_{\Omega}^J} \right) + L \left( \prod_{J=1}^{\alpha} (\mathfrak{M}_{1J}^I)^{z_{\Omega}^J} \right), \\ \left( -\prod_{J=1}^{\alpha} (|\mathfrak{N}_{1J}^R|)^{z_{\Omega}^J} \right) + L \left( -\prod_{J=1}^{\alpha} (|\mathfrak{N}_{1J}^I|)^{z_{\Omega}^J} \right). \end{array} \right)$$

$$\begin{aligned}
 &= \left( \begin{array}{l} \left( 1 - \prod_{J=1}^{\varkappa} \left( \prod_{I=1}^1 (1 - \mathfrak{C}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( 1 - \prod_{J=1}^{\varkappa} \left( \prod_{I=1}^1 (1 - \mathfrak{C}_{IJ}^L)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( -1 + \prod_{J=1}^{\varkappa} \left( \prod_{I=1}^1 (1 + \mathfrak{D}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( -1 + \prod_{J=1}^{\varkappa} \left( \prod_{I=1}^1 (1 + \mathfrak{D}_{IJ}^L)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( \prod_{J=1}^{\varkappa} \left( \prod_{I=1}^1 (\mathfrak{M}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( \prod_{J=1}^{\varkappa} \left( \prod_{I=1}^1 (\mathfrak{M}_{IJ}^L)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( -\prod_{J=1}^{\varkappa} \left( \prod_{I=1}^1 (|\mathfrak{N}_{IJ}^R|)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( -\prod_{J=1}^{\varkappa} \left( \prod_{I=1}^1 (|\mathfrak{N}_{IJ}^L|)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right). \end{array} \right) \\
 &= \bigoplus_{J=1}^{\varkappa} z_{\Omega}^J \left( \bigoplus_{I=1}^1 z_{\psi}^I \mathbb{A}_{BC}^{IJ} \right) = BCIFSWA \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\varkappa \varkappa} \right)
 \end{aligned}$$

But, if we consider  $\varkappa = 1$  and  $z_{\Omega}^J = 1$ , then

$$\begin{aligned}
 BCIFSWA \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\check{\varkappa} 1} \right) &= \left( \bigoplus_{I=1}^{\check{\varkappa}} z_{\psi}^I \mathbb{A}_{BC}^{I1} \right) \\
 &= \left( \begin{array}{l} \left( 1 - \prod_{I=1}^{\check{\varkappa}} (1 - \mathfrak{C}_{IJ}^R)^{z_{\psi}^I} \right) + L \left( 1 - \prod_{I=1}^{\check{\varkappa}} (1 - \mathfrak{C}_{IJ}^L)^{z_{\psi}^I} \right), \\ \left( -1 + \left( \prod_{I=1}^{\check{\varkappa}} (1 + \mathfrak{D}_{IJ}^R)^{z_{\psi}^I} \right) \right) + L \left( -1 + \left( \prod_{I=1}^{\check{\varkappa}} (1 + \mathfrak{D}_{IJ}^L)^{z_{\psi}^I} \right) \right), \\ \left( \prod_{I=1}^{\check{\varkappa}} (\mathfrak{M}_{IJ}^R)^{z_{\psi}^I} \right) + L \left( \prod_{I=1}^{\check{\varkappa}} (\mathfrak{M}_{IJ}^L)^{z_{\psi}^I} \right), \\ \left( -\prod_{I=1}^{\check{\varkappa}} (|\mathfrak{N}_{IJ}^R|)^{z_{\psi}^I} \right) + L \left( -\prod_{I=1}^{\check{\varkappa}} (|\mathfrak{N}_{IJ}^L|)^{z_{\psi}^I} \right). \end{array} \right) \\
 &= \left( \begin{array}{l} \left( 1 - \prod_{J=1}^1 \left( \prod_{I=1}^{\check{\varkappa}} (1 - \mathfrak{C}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( 1 - \prod_{J=1}^1 \left( \prod_{I=1}^{\check{\varkappa}} (1 - \mathfrak{C}_{IJ}^L)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( -1 + \prod_{J=1}^1 \left( \prod_{I=1}^{\check{\varkappa}} (1 + \mathfrak{D}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( -1 + \prod_{J=1}^1 \left( \prod_{I=1}^{\check{\varkappa}} (1 + \mathfrak{D}_{IJ}^L)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( \prod_{J=1}^1 \left( \prod_{I=1}^{\check{\varkappa}} (\mathfrak{M}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( \prod_{J=1}^1 \left( \prod_{I=1}^{\check{\varkappa}} (\mathfrak{M}_{IJ}^L)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( -\prod_{J=1}^1 \left( \prod_{I=1}^{\check{\varkappa}} (|\mathfrak{N}_{IJ}^R|)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( -\prod_{J=1}^1 \left( \prod_{I=1}^{\check{\varkappa}} (|\mathfrak{N}_{IJ}^L|)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right). \end{array} \right) \\
 &= \bigoplus_{J=1}^1 z_{\Omega}^J \left( \bigoplus_{I=1}^{\check{\varkappa}} z_{\psi}^I \mathbb{A}_{BC}^{IJ} \right) = BCIFSWA \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\check{\varkappa} \varkappa} \right)
 \end{aligned}$$

The proposed theory is held for  $\tilde{\alpha} = \varkappa = 1$ . Additionally, We consider  $\varkappa = \sigma_1 + 1, \tilde{\alpha} = \sigma_2$  and  $\varkappa = \sigma_1, \tilde{\alpha} = \sigma_2 + 1$ , thus

$$\begin{aligned} & \bigoplus_{J=1}^{\sigma_1+1} \mathcal{Z}_{\Omega}^J \left( \bigoplus_{I=1}^{\sigma_2} \mathcal{Z}_{\psi}^I \mathbb{A}_{BC}^{IJ} \right) \\ &= \left( \begin{aligned} & \left( 1 - \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (1 - \mathfrak{C}_{IJ}^R)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right) + \mathbb{L} \left( 1 - \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (1 - \mathfrak{C}_{IJ}^I)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right), \\ & \left( -1 + \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (1 + \mathfrak{D}_{IJ}^R)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right) + \mathbb{L} \left( -1 + \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (1 + \mathfrak{D}_{IJ}^I)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right), \\ & \left( \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (\mathfrak{M}_{IJ}^R)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right) + \mathbb{L} \left( \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (\mathfrak{M}_{IJ}^I)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right), \\ & \left( -\prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (\mathfrak{N}_{IJ}^R)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right) + \mathbb{L} \left( -\prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (\mathfrak{N}_{IJ}^I)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right). \end{aligned} \right) \end{aligned}$$

and

$$\begin{aligned} & \bigoplus_{J=1}^{\sigma_1} \mathcal{Z}_{\Omega}^J \left( \bigoplus_{I=1}^{\sigma_2+1} \mathcal{Z}_{\psi}^I \mathbb{A}_{BC}^{IJ} \right) \\ &= \left( \begin{aligned} & \left( 1 - \prod_{J=1}^{\sigma_1} \left( \prod_{I=1}^{\sigma_2+1} (1 - \mathfrak{C}_{IJ}^R)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right) + \mathbb{L} \left( 1 - \prod_{J=1}^{\sigma_1} \left( \prod_{I=1}^{\sigma_2+1} (1 - \mathfrak{C}_{IJ}^I)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right), \\ & \left( -1 + \prod_{J=1}^{\sigma_1} \left( \prod_{I=1}^{\sigma_2+1} (1 + \mathfrak{D}_{IJ}^R)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right) + \mathbb{L} \left( -1 + \prod_{J=1}^{\sigma_1} \left( \prod_{I=1}^{\sigma_2+1} (1 + \mathfrak{D}_{IJ}^I)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right), \\ & \left( \prod_{J=1}^{\sigma_1} \left( \prod_{I=1}^{\sigma_2+1} (\mathfrak{M}_{IJ}^R)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right) + \mathbb{L} \left( \prod_{J=1}^{\sigma_1} \left( \prod_{I=1}^{\sigma_2+1} (\mathfrak{M}_{IJ}^I)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right), \\ & \left( -\prod_{J=1}^{\sigma_1} \left( \prod_{I=1}^{\sigma_2+1} (\mathfrak{N}_{IJ}^R)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right) + \mathbb{L} \left( -\prod_{J=1}^{\sigma_1} \left( \prod_{I=1}^{\sigma_2+1} (\mathfrak{N}_{IJ}^I)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right). \end{aligned} \right) \end{aligned}$$

Thus, we prove it for  $\varkappa = \sigma_1 + 1$  and  $\tilde{\alpha} = \sigma_2 + 1$ , such as

$$\begin{aligned} \bigoplus_{J=1}^{\sigma_1+1} \mathcal{Z}_{\Omega}^J \left( \bigoplus_{I=1}^{\sigma_2+1} \mathcal{Z}_{\psi}^I \mathbb{A}_{BC}^{IJ} \right) &= \bigoplus_{J=1}^{\sigma_1+1} \mathcal{Z}_{\Omega}^J \left( \bigoplus_{I=1}^{\sigma_2} \mathcal{Z}_{\psi}^I \mathbb{A}_{BC}^{IJ} \oplus \mathcal{Z}_{\psi}^{\sigma_2+1} \mathbb{A}_{BC}^{(\sigma_2+1)J} \right) \\ &= \bigoplus_{J=1}^{\sigma_1+1} \mathcal{Z}_{\Omega}^J \left( \bigoplus_{I=1}^{\sigma_2} \mathcal{Z}_{\psi}^I \mathbb{A}_{BC}^{IJ} \right) \oplus \bigoplus_{J=1}^{\sigma_1+1} \mathcal{Z}_{\Omega}^J \left( \mathcal{Z}_{\psi}^{\sigma_2+1} \mathbb{A}_{BC}^{(\sigma_2+1)J} \right) \end{aligned}$$

$$= \left( \begin{array}{l} \left( 1 - \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (1 - \mathfrak{C}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( 1 - \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (1 - \mathfrak{C}_{IJ}^I)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( -1 + \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (1 + \mathfrak{D}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( -1 + \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (1 + \mathfrak{D}_{IJ}^I)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (\mathfrak{M}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (\mathfrak{M}_{IJ}^I)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( -\prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (\mathfrak{N}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( -\prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (\mathfrak{N}_{IJ}^I)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right). \end{array} \right)$$

$$\oplus \left( \begin{array}{l} \left( 1 - \prod_{J=1}^{\sigma_1+1} \left( (1 - \mathfrak{C}_{(\sigma_2+1)J}^R)^{z_{\psi}^{(\sigma_2+1)}} \right)^{z_{\Omega}^J} \right) + \\ \mathbb{L} \left( 1 - \prod_{J=1}^{\sigma_1+1} \left( (1 - \mathfrak{C}_{(\sigma_2+1)J}^I)^{z_{\psi}^{(\sigma_2+1)}} \right)^{z_{\Omega}^J} \right), \\ \left( -1 + \prod_{J=1}^{\sigma_1+1} \left( (1 + \mathfrak{D}_{(\sigma_2+1)J}^R)^{z_{\psi}^{(\sigma_2+1)}} \right)^{z_{\Omega}^J} \right) + \\ \mathbb{L} \left( -1 + \prod_{J=1}^{\sigma_1+1} \left( (1 + \mathfrak{D}_{(\sigma_2+1)J}^I)^{z_{\psi}^{(\sigma_2+1)}} \right)^{z_{\Omega}^J} \right), \\ \left( \prod_{J=1}^{\sigma_1+1} \left( (\mathfrak{M}_{(\sigma_2+1)J}^R)^{z_{\psi}^{(\sigma_2+1)}} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( \prod_{J=1}^{\sigma_1+1} \left( (\mathfrak{M}_{(\sigma_2+1)J}^I)^{z_{\psi}^{(\sigma_2+1)}} \right)^{z_{\Omega}^J} \right), \\ \left( -\prod_{J=1}^{\sigma_1+1} \left( (\mathfrak{N}_{(\sigma_2+1)J}^R)^{z_{\psi}^{(\sigma_2+1)}} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( -\prod_{J=1}^{\sigma_1+1} \left( (\mathfrak{N}_{(\sigma_2+1)J}^I)^{z_{\psi}^{(\sigma_2+1)}} \right)^{z_{\Omega}^J} \right). \end{array} \right)$$

$$= \left( \begin{aligned} & \left( 1 - \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2+1} (1 - \mathfrak{C}_{IJ}^R)^{z_{\Omega}^I} \right) \right) + \\ & \mathbb{L} \left( 1 - \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2+1} (1 - \mathfrak{C}_{IJ}^I)^{z_{\Omega}^I} \right) \right), \\ & \left( -1 + \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2+1} (1 + \mathfrak{D}_{IJ}^R)^{z_{\Omega}^I} \right) \right) + \\ & \mathbb{L} \left( -1 + \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2+1} (1 + \mathfrak{D}_{IJ}^I)^{z_{\Omega}^I} \right) \right), \\ & \left( \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2+1} (\mathfrak{M}_{IJ}^R)^{z_{\Omega}^I} \right) \right) + \mathbb{L} \left( \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2+1} (\mathfrak{M}_{IJ}^I)^{z_{\Omega}^I} \right) \right), \\ & \left( -\prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2+1} (\mathfrak{N}_{IJ}^R)^{z_{\Omega}^I} \right) \right) + \mathbb{L} \left( -\prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2+1} (\mathfrak{N}_{IJ}^I)^{z_{\Omega}^I} \right) \right). \end{aligned} \right)$$

The proposed theory is hold for  $\alpha = \sigma_1 + 1$  and  $\tilde{\alpha} = \sigma_2 + 1$ . Further, we evaluate the basic properties of the above theory.

Property 1: When  $\mathbb{A}_{BC}^{IJ} = \mathbb{A}_{BC} = (\mathfrak{C}^R + \mathbb{L}\mathfrak{C}^I, \mathfrak{D}^R + \mathbb{L}\mathfrak{D}^I, \mathfrak{M}^R + \mathbb{L}\mathfrak{M}^I, \mathfrak{N}^R + \mathbb{L}\mathfrak{N}^I)$ ,  $I, J = 1, 2, \dots, \tilde{\alpha}\alpha$ , thus  $BCIFSWA(\mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}\alpha}) = \mathbb{A}_{BC}$ . (Idempotency)

Property 2: When  $\mathbb{A}_{BC}^{IJ} \leq \mathbb{A}_{BC}^{*IJ}$ , thus  $BCIFSWA(\mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}\alpha}) \leq BCIFSWA(\mathbb{A}_{BC}^{*11}, \mathbb{A}_{BC}^{*12}, \dots, \mathbb{A}_{BC}^{*\tilde{\alpha}\alpha})$ . (Monotonicity)

Property 3: When  $\mathbb{A}_{BC}^- = \left( \min_I \min_J \mathfrak{C}_{IJ}^R + \mathbb{L} \left( \min_I \min_J \mathfrak{C}_{IJ}^I \right), \min_I \min_J \mathfrak{D}_{IJ}^R + \mathbb{L} \left( \min_I \min_J \mathfrak{D}_{IJ}^I \right), \max_I \max_J \mathfrak{M}_{IJ}^R + \mathbb{L} \left( \max_I \max_J \mathfrak{M}_{IJ}^I \right), \max_I \max_J \mathfrak{N}_{IJ}^R + \mathbb{L} \left( \max_I \max_J \mathfrak{N}_{IJ}^I \right) \right)$ , and  $\mathbb{A}_{BC}^+ = \left( \max_I \max_J \mathfrak{C}_{IJ}^R + \mathbb{L} \left( \max_I \max_J \mathfrak{C}_{IJ}^I \right), \max_I \max_J \mathfrak{D}_{IJ}^R + \mathbb{L} \left( \max_I \max_J \mathfrak{D}_{IJ}^I \right), \min_I \min_J \mathfrak{M}_{IJ}^R + \mathbb{L} \left( \min_I \min_J \mathfrak{M}_{IJ}^I \right), \min_I \min_J \mathfrak{N}_{IJ}^R + \mathbb{L} \left( \min_I \min_J \mathfrak{N}_{IJ}^I \right) \right)$ ,  $I, J = 1, 2, \dots, \tilde{\alpha}\alpha$ , thus  $\mathbb{A}_{BC}^- \leq BCIFSWA(\mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}\alpha}) \leq \mathbb{A}_{BC}^+$ . (Boundedness)

Proof: Let  $\mathbb{A}_{BC}^- = \left( \min_I \min_J \mathbb{C}_{IJ}^R + L \left( \min_I \min_J \mathbb{C}_{IJ}^L \right), \min_I \min_J \mathbb{D}_{IJ}^R + L \left( \min_I \min_J \mathbb{D}_{IJ}^L \right), \max_I \max_J \mu_{IJ}^R + L \left( \max_I \max_J \mu_{IJ}^L \right), \max_I \max_J \eta_{IJ}^R + L \left( \max_I \max_J \eta_{IJ}^L \right) \right)$  and  $\mathbb{A}_{BC}^+ = \left( \max_I \max_J \mathbb{C}_{IJ}^R + L \left( \max_I \max_J \mathbb{C}_{IJ}^L \right), \max_I \max_J \mathbb{D}_{IJ}^R + L \left( \max_I \max_J \mathbb{D}_{IJ}^L \right), \min_I \min_J \mu_{IJ}^R + L \left( \min_I \min_J \mu_{IJ}^L \right), \min_I \min_J \eta_{IJ}^R + L \left( \min_I \min_J \eta_{IJ}^L \right) \right)$ ,  $I, J = 1, 2, \dots, \tilde{\alpha}_A$ , Thus

$$BCIFSWA \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}_A} \right) \leq BCIFSWA \left( \mathbb{A}_{BC}^{+11}, \mathbb{A}_{BC}^{+12}, \dots, \mathbb{A}_{BC}^{+\tilde{\alpha}_A} \right) = \mathbb{A}_{BC}^+$$

$$BCIFSWA \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}_A} \right) \geq BCIFSWA \left( \mathbb{A}_{BC}^{-11}, \mathbb{A}_{BC}^{-12}, \dots, \mathbb{A}_{BC}^{-\tilde{\alpha}_A} \right) = \mathbb{A}_{BC}^-$$

Thus

$$\mathbb{A}_{BC}^- \leq BCIFSWA \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}_A} \right) \leq \mathbb{A}_{BC}^+$$

Definition 5: Consider any family of BCIFSNs  $\mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}_A}$ . Thus, the technique of BCIFSOWA operator based on  $\mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}_A}$  is mentioned and developed by:

$$BCIFSOWA \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}_A} \right) = \bigoplus_{J=1}^{\tilde{\alpha}_A} \mathcal{Z}_{\Omega}^J \left( \bigoplus_{I=1}^{\tilde{\alpha}_A} \mathcal{Z}_{\psi}^I \mathbb{A}_{BC}^{o(I)o(J)} \right)$$

The mathematical notion of the weight vector is as follows, such as  $\mathcal{Z}_{\Omega}^J, \mathcal{Z}_{\psi}^I \in [0, 1]$  with  $\sum_{I=1}^{\tilde{\alpha}_A} \mathcal{Z}_{\psi}^I = 1$  and  $\sum_{J=1}^{\tilde{\alpha}_A} \mathcal{Z}_{\Omega}^J = 1$  and  $o(I), o(J) \leq o(I - 1), o(J - 1)$ .

Theorem 2: Prove that the aggregated value of the  $BCIFSOWA \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}_A} \right)$  is also a BCIFSN, such as

$$BCIFSOWA \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}^{\mathfrak{A}}} \right)$$

$$= \left( \begin{array}{l} \left( 1 - \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}^{\mathfrak{A}}} (1 - \mathfrak{C}_{o(I)o(J)}^R)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right) + \\ \mathbb{L} \left( 1 - \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}^{\mathfrak{A}}} (1 - \mathfrak{C}_{o(I)o(J)}^I)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right), \\ \left( -1 + \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}^{\mathfrak{A}}} (|1 + \mathfrak{D}_{o(I)o(J)}^R|)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right) + \\ \mathbb{L} \left( -1 + \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}^{\mathfrak{A}}} (|1 + \mathfrak{D}_{o(I)o(J)}^I|)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right), \\ \left( \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}^{\mathfrak{A}}} (\mathfrak{M}_{o(I)o(J)}^R)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right) + \mathbb{L} \left( \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}^{\mathfrak{A}}} (\mathfrak{M}_{o(I)o(J)}^I)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right), \\ \left( -\prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}^{\mathfrak{A}}} (|\mathfrak{N}_{o(I)o(J)}^R|)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right) + \mathbb{L} \left( -\prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}^{\mathfrak{A}}} (|\mathfrak{N}_{o(I)o(J)}^I|)^{2_{\psi}^I} \right)^{2_{\Omega}^J} \right). \end{array} \right)$$

Property 4: When  $\mathbb{A}_{BC}^{IJ} = \mathbb{A}_{BC} = (\mathfrak{C}^R + \mathbb{L}\mathfrak{C}^I, \mathfrak{D}^R + \mathbb{L}\mathfrak{D}^I, \mathfrak{M}^R + \mathbb{L}\mathfrak{M}^I, \mathfrak{N}^R + \mathbb{L}\mathfrak{N}^I)$ ,  $I, J = 1, 2, \dots, \tilde{\alpha}^{\mathfrak{A}}$ , thus  $BCIFSOWA \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}^{\mathfrak{A}}} \right) = \mathbb{A}_{BC}$ . (Idempotency)

Property 5: When  $\mathbb{A}_{BC}^{IJ} \leq \mathbb{A}_{BC}^{*IJ}$ , thus  $BCIFSOWA \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}^{\mathfrak{A}}} \right) \leq BCIFSOWA \left( \mathbb{A}_{BC}^{*11}, \mathbb{A}_{BC}^{*12}, \dots, \mathbb{A}_{BC}^{*\tilde{\alpha}^{\mathfrak{A}}} \right)$ . (Monotonicity)

Property 6: When  $\mathbb{A}_{BC}^- = \left( \min_I \min_J \mathfrak{C}_{IJ}^R + \mathbb{L} \left( \min_I \min_J \mathfrak{C}_{IJ}^I \right), \min_I \min_J \mathfrak{D}_{IJ}^R + \mathbb{L} \left( \min_I \min_J \mathfrak{D}_{IJ}^I \right), \max_I \max_J \mathfrak{M}_{IJ}^R + \mathbb{L} \left( \max_I \max_J \mathfrak{M}_{IJ}^I \right), \max_I \max_J \mathfrak{N}_{IJ}^R + \mathbb{L} \left( \max_I \max_J \mathfrak{N}_{IJ}^I \right) \right)$ , and  $\mathbb{A}_{BC}^+ = \left( \max_I \max_J \mathfrak{C}_{IJ}^R + \mathbb{L} \left( \max_I \max_J \mathfrak{C}_{IJ}^I \right), \max_I \max_J \mathfrak{D}_{IJ}^R + \mathbb{L} \left( \max_I \max_J \mathfrak{D}_{IJ}^I \right), \min_I \min_J \mathfrak{M}_{IJ}^R + \mathbb{L} \left( \min_I \min_J \mathfrak{M}_{IJ}^I \right), \min_I \min_J \mathfrak{N}_{IJ}^R + \mathbb{L} \left( \min_I \min_J \mathfrak{N}_{IJ}^I \right) \right)$ ,  $I, J = 1, 2, \dots, \tilde{\alpha}^{\mathfrak{A}}$ , thus  $\mathbb{A}_{BC}^- \leq BCIFSOWA \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}^{\mathfrak{A}}} \right) \leq \mathbb{A}_{BC}^+$ . (Boundedness)

Definition 6: Consider any family of BCIFSNs  $\mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}^{\mathfrak{A}}}$ . Thus, the technique of BCIFSWG operator based on  $\mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}^{\mathfrak{A}}}$  is mentioned and developed by:

$$BCIFSWG \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}^{\mathfrak{A}}} \right) = \otimes_{J=1}^{\mathfrak{A}} \left( \otimes_{I=1}^{\tilde{\alpha}^{\mathfrak{A}}} \left( \mathbb{A}_{BC}^{IJ} \right)^{2_{\psi}^I} \right)^{2_{\Omega}^J}$$

The mathematical notion of the weight vector is as follows, such as  $z_{\Omega}^J, z_{\psi}^I \in [0, 1]$  with  $\sum_{I=1}^{\tilde{\alpha}} z_{\psi}^I = 1$  and  $\sum_{J=1}^{\mathfrak{A}} z_{\Omega}^J = 1$ .

Theorem 3: Prove that the aggregated value of the  $BCIFSWG(\mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}\mathfrak{A}})$  is also a BCIFSN, such as

$$BCIFSWG(\mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}\mathfrak{A}}) = \left( \begin{array}{l} \left( \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}} (\mathbb{C}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}} (\mathbb{C}_{IJ}^I)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( - \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}} (\mathbb{D}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( - \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}} (\mathbb{D}_{IJ}^I)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( 1 - \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}} (1 - \mathfrak{m}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( 1 - \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}} (1 - \mathfrak{m}_{IJ}^I)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( -1 + \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}} (1 + \mathfrak{n}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( -1 + \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}} (1 + \mathfrak{n}_{IJ}^I)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right). \end{array} \right)$$

Proof: Using the mathematical induction, we prove the above information. Let  $\tilde{\alpha} = 1$  and  $z_{\psi}^I = 1$ , thus

$$BCIFSWG(\mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{1\mathfrak{A}}) = \otimes_{J=1}^{\mathfrak{A}} (\mathbb{A}_{BC}^{1J})^{z_{\Omega}^J} = \left( \begin{array}{l} \left( \prod_{J=1}^{\mathfrak{A}} (\mathbb{C}_{IJ}^R)^{z_{\Omega}^J} \right) + \mathbb{L} \left( \prod_{J=1}^{\mathfrak{A}} (\mathbb{C}_{IJ}^I)^{z_{\Omega}^J} \right), \\ \left( - \prod_{J=1}^{\mathfrak{A}} (\mathbb{D}_{IJ}^R)^{z_{\Omega}^J} \right) + \mathbb{L} \left( - \prod_{J=1}^{\mathfrak{A}} (\mathbb{D}_{IJ}^I)^{z_{\Omega}^J} \right), \\ \left( 1 - \prod_{J=1}^{\mathfrak{A}} (1 - \mathfrak{m}_{IJ}^R)^{z_{\Omega}^J} \right) + \mathbb{L} \left( 1 - \prod_{J=1}^{\mathfrak{A}} (1 - \mathfrak{m}_{IJ}^I)^{z_{\Omega}^J} \right), \\ \left( -1 + \prod_{J=1}^{\mathfrak{A}} (1 + \mathfrak{n}_{IJ}^R)^{z_{\Omega}^J} \right) + \mathbb{L} \left( -1 + \prod_{J=1}^{\mathfrak{A}} (1 + \mathfrak{n}_{IJ}^I)^{z_{\Omega}^J} \right). \end{array} \right)$$

$$\begin{aligned}
 &= \left( \begin{array}{l} \left( \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^1 (\mathfrak{C}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^1 (\mathfrak{C}_{IJ}^L)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( - \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^1 (\mathfrak{D}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( - \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^1 (\mathfrak{D}_{IJ}^L)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( 1 - \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^1 (1 - \mathfrak{M}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( 1 - \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^1 (1 - \mathfrak{M}_{IJ}^L)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( -1 + \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^1 (1 + \mathfrak{N}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( -1 + \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^1 (1 + \mathfrak{N}_{IJ}^L)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right). \end{array} \right) \\
 &= \otimes_{J=1}^{\mathfrak{A}} \left( \otimes_{I=1}^1 (\mathfrak{A}_{BC}^{IJ})^{z_{\psi}^I} \right)^{z_{\Omega}^J} = BCIFSWG \left( \mathfrak{A}_{BC}^{11}, \mathfrak{A}_{BC}^{12}, \dots, \mathfrak{A}_{BC}^{\check{\alpha}\mathfrak{A}} \right)
 \end{aligned}$$

If  $\mathfrak{A} = 1$  and  $z_{\Omega}^J = 1$ , then

$$\begin{aligned}
 BCIFSWG \left( \mathfrak{A}_{BC}^{11}, \mathfrak{A}_{BC}^{12}, \dots, \mathfrak{A}_{BC}^{\check{\alpha}1} \right) &= \left( \otimes_{I=1}^{\check{\alpha}} (\mathfrak{A}_{BC}^{I1})^{z_{\psi}^I} \right) \\
 &= \left( \begin{array}{l} \left( \prod_{I=1}^{\check{\alpha}} (\mathfrak{C}_{IJ}^R)^{z_{\psi}^I} \right) + \mathbb{L} \left( \prod_{I=1}^{\check{\alpha}} (\mathfrak{C}_{IJ}^L)^{z_{\psi}^I} \right), \\ \left( - \prod_{I=1}^{\check{\alpha}} (\mathfrak{D}_{IJ}^R)^{z_{\psi}^I} \right) + \mathbb{L} \left( - \prod_{I=1}^{\check{\alpha}} (\mathfrak{D}_{IJ}^L)^{z_{\psi}^I} \right), \\ \left( 1 - \prod_{I=1}^{\check{\alpha}} (1 - \mathfrak{M}_{IJ}^R)^{z_{\psi}^I} \right) + \mathbb{L} \left( 1 - \prod_{I=1}^{\check{\alpha}} (1 - \mathfrak{M}_{IJ}^L)^{z_{\psi}^I} \right), \\ \left( -1 + \left( \prod_{I=1}^{\check{\alpha}} (1 + \mathfrak{N}_{IJ}^R)^{z_{\psi}^I} \right) \right) + \mathbb{L} \left( -1 + \left( \prod_{I=1}^{\check{\alpha}} (1 + \mathfrak{N}_{IJ}^L)^{z_{\psi}^I} \right) \right). \end{array} \right) \\
 &= \left( \begin{array}{l} \left( \prod_{J=1}^1 \left( \prod_{I=1}^{\check{\alpha}} (\mathfrak{C}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( \prod_{J=1}^1 \left( \prod_{I=1}^{\check{\alpha}} (\mathfrak{C}_{IJ}^L)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( - \prod_{J=1}^1 \left( \prod_{I=1}^{\check{\alpha}} (\mathfrak{D}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( - \prod_{J=1}^1 \left( \prod_{I=1}^{\check{\alpha}} (\mathfrak{D}_{IJ}^L)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( 1 - \prod_{J=1}^1 \left( \prod_{I=1}^{\check{\alpha}} (1 - \mathfrak{M}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( 1 - \prod_{J=1}^1 \left( \prod_{I=1}^{\check{\alpha}} (1 - \mathfrak{M}_{IJ}^L)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( -1 + \prod_{J=1}^1 \left( \prod_{I=1}^{\check{\alpha}} (1 + \mathfrak{N}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( -1 + \prod_{J=1}^1 \left( \prod_{I=1}^{\check{\alpha}} (1 + \mathfrak{N}_{IJ}^L)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right). \end{array} \right)
 \end{aligned}$$

$$= \otimes_{J=1}^1 \left( \otimes_{I=1}^{\tilde{\alpha}} (\mathbb{A}_{BC}^{IJ})^{z_{\psi}^I} \right)^{z_{\Omega}^J} = BCIFSWG \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}\alpha} \right)$$

The proposed theory is held for  $\tilde{\alpha} = \alpha = 1$ . Additionally, if  $\alpha = \sigma_1 + 1$ ,  $\tilde{\alpha} = \sigma_2$  and  $\alpha = \sigma_1$ ,  $\tilde{\alpha} = \sigma_2 + 1$ , thus

$$\otimes_{J=1}^{\sigma_1+1} \left( \otimes_{I=1}^{\sigma_2} (\mathbb{A}_{BC}^{IJ})^{z_{\psi}^I} \right)^{z_{\Omega}^J} = \left( \begin{array}{l} \left( \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (\mathbb{C}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (\mathbb{C}_{IJ}^L)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( - \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (|\mathcal{D}_{IJ}^R|)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( - \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (|\mathcal{D}_{IJ}^L|)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( 1 - \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (1 - \mathfrak{M}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( 1 - \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (1 - \mathfrak{M}_{IJ}^L)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( -1 + \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (1 + \mathfrak{N}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( -1 + \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (1 + \mathfrak{N}_{IJ}^L)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right). \end{array} \right)$$

and

$$\otimes_{J=1}^{\sigma_1} \left( \otimes_{I=1}^{\sigma_2+1} (\mathbb{A}_{BC}^{IJ})^{z_{\psi}^I} \right)^{z_{\Omega}^J} = \left( \begin{array}{l} \left( \prod_{J=1}^{\sigma_1} \left( \prod_{I=1}^{\sigma_2+1} (\mathbb{C}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( \prod_{J=1}^{\sigma_1} \left( \prod_{I=1}^{\sigma_2+1} (\mathbb{C}_{IJ}^L)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( - \prod_{J=1}^{\sigma_1} \left( \prod_{I=1}^{\sigma_2+1} (|\mathcal{D}_{IJ}^R|)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( - \prod_{J=1}^{\sigma_1} \left( \prod_{I=1}^{\sigma_2+1} (|\mathcal{D}_{IJ}^L|)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( 1 - \prod_{J=1}^{\sigma_1} \left( \prod_{I=1}^{\sigma_2+1} (1 - \mathfrak{M}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( 1 - \prod_{J=1}^{\sigma_1} \left( \prod_{I=1}^{\sigma_2+1} (1 - \mathfrak{M}_{IJ}^L)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( -1 + \prod_{J=1}^{\sigma_1} \left( \prod_{I=1}^{\sigma_2+1} (1 + \mathfrak{N}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( -1 + \prod_{J=1}^{\sigma_1} \left( \prod_{I=1}^{\sigma_2+1} (1 + \mathfrak{N}_{IJ}^L)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right). \end{array} \right)$$

Thus, we prove it for  $\alpha = \sigma_1 + 1$  and  $\tilde{\alpha} = \sigma_2 + 1$ , such as

$$\begin{aligned} \otimes_{J=1}^{\sigma_1+1} \left( \otimes_{I=1}^{\sigma_2+1} (\mathbb{A}_{BC}^{IJ})^{z_{\psi}^I} \right)^{z_{\Omega}^J} &= \otimes_{J=1}^{\sigma_1+1} \left( \otimes_{I=1}^{\sigma_2+1} (\mathbb{A}_{BC}^{IJ})^{z_{\psi}^I} \otimes (\mathbb{A}_{BC}^{(\sigma_2+1)J})^{z_{\psi}^{\sigma_2+1}} \right)^{z_{\Omega}^J} \\ &= \otimes_{J=1}^{\sigma_1+1} \left( \otimes_{I=1}^{\sigma_2} (\mathbb{A}_{BC}^{IJ})^{z_{\psi}^I} \right)^{z_{\Omega}^J} \otimes_{J=1}^{\sigma_1+1} \left( (\mathbb{A}_{BC}^{(\sigma_2+1)J})^{z_{\psi}^{\sigma_2+1}} \right)^{z_{\Omega}^J} \end{aligned}$$

$$= \left( \begin{array}{l} \left( \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (\mathbb{C}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (\mathbb{C}_{IJ}^I)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( - \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (|\mathbb{D}_{IJ}^R|)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( - \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (|\mathbb{D}_{IJ}^I|)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( 1 - \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (1 - \mathbb{M}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( 1 - \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (1 - \mathbb{M}_{IJ}^I)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( -1 + \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (1 + \mathbb{N}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( -1 + \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2} (1 + \mathbb{N}_{IJ}^I)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right). \end{array} \right)$$

$$\oplus \left( \begin{array}{l} \left( \prod_{J=1}^{\sigma_1+1} \left( (\mathbb{C}_{(\sigma_2+1)J}^R)^{z_{\psi}^{(\sigma_2+1)}} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( \prod_{J=1}^{\sigma_1+1} \left( (\mathbb{C}_{(\sigma_2+1)J}^I)^{z_{\psi}^{(\sigma_2+1)}} \right)^{z_{\Omega}^J} \right), \\ \left( - \prod_{J=1}^{\sigma_1+1} \left( (|\mathbb{D}_{(\sigma_2+1)J}^R|)^{z_{\psi}^{(\sigma_2+1)}} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( - \prod_{J=1}^{\sigma_1+1} \left( (|\mathbb{D}_{(\sigma_2+1)J}^I|)^{z_{\psi}^{(\sigma_2+1)}} \right)^{z_{\Omega}^J} \right), \\ \left( 1 - \prod_{J=1}^{\sigma_1+1} \left( (1 - \mathbb{M}_{(\sigma_2+1)J}^R)^{z_{\psi}^{(\sigma_2+1)}} \right)^{z_{\Omega}^J} \right) + \\ \mathbb{L} \left( 1 - \prod_{J=1}^{\sigma_1+1} \left( (1 - \mathbb{M}_{(\sigma_2+1)J}^I)^{z_{\psi}^{(\sigma_2+1)}} \right)^{z_{\Omega}^J} \right), \\ \left( -1 + \prod_{J=1}^{\sigma_1+1} \left( (1 + \mathbb{N}_{(\sigma_2+1)J}^R)^{z_{\psi}^{(\sigma_2+1)}} \right)^{z_{\Omega}^J} \right) + \\ \mathbb{L} \left( -1 + \prod_{J=1}^{\sigma_1+1} \left( (1 + \mathbb{N}_{(\sigma_2+1)J}^I)^{z_{\psi}^{(\sigma_2+1)}} \right)^{z_{\Omega}^J} \right). \end{array} \right)$$

$$= \left( \begin{array}{l} \left( \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2+1} (\mathcal{C}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathcal{L} \left( \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2+1} (\mathcal{C}_{IJ}^I)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( - \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2+1} (\mathcal{D}_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathcal{L} \left( - \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2+1} (\mathcal{D}_{IJ}^I)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( 1 - \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2+1} (1 - \mu_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \mathcal{L} \left( 1 - \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2+1} (1 - \mu_{IJ}^I)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( -1 + \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2+1} (1 + \eta_{IJ}^R)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \\ \mathcal{L} \left( -1 + \prod_{J=1}^{\sigma_1+1} \left( \prod_{I=1}^{\sigma_2+1} (1 + \eta_{IJ}^I)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right). \end{array} \right)$$

The proposed theory is hold for  $\alpha = \sigma_1 + 1$  and  $\tilde{\alpha} = \sigma_2 + 1$ .

Property 7: When  $\mathbb{A}_{BC}^{IJ} = \mathbb{A}_{BC} = (\mathcal{C}^R + \mathcal{L}\mathcal{C}^I, \mathcal{D}^R + \mathcal{L}\mathcal{D}^I, \mu^R + \mathcal{L}\mu^I, \eta^R + \mathcal{L}\eta^I)$ ,  $I, J = 1, 2, \dots, \tilde{\alpha}$ , thus  $BCIFSWG(\mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}\alpha}) = \mathbb{A}_{BC}$ . (Idempotency)

Property 8: When  $\mathbb{A}_{BC}^{IJ} \leq \mathbb{A}_{BC}^{*IJ}$ , thus  $BCIFSWG(\mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}\alpha}) \leq BCIFSWG(\mathbb{A}_{BC}^{*11}, \mathbb{A}_{BC}^{*12}, \dots, \mathbb{A}_{BC}^{*\tilde{\alpha}\alpha})$ . (Monotonicity)

Property 9: When  $\mathbb{A}_{BC}^- = \left( \min_I \min_J \mathcal{C}_{IJ}^R + \mathcal{L}(\min_I \min_J \mathcal{C}_{IJ}^I), \min_I \min_J \mathcal{D}_{IJ}^R + \mathcal{L}(\min_I \min_J \mathcal{D}_{IJ}^I), \max_I \max_J \mu_{IJ}^R + \mathcal{L}(\max_I \max_J \mu_{IJ}^I), \max_I \max_J \eta_{IJ}^R + \mathcal{L}(\max_I \max_J \eta_{IJ}^I) \right)$ , and  $\mathbb{A}_{BC}^+ = \left( \max_I \max_J \mathcal{C}_{IJ}^R + \mathcal{L}(\max_I \max_J \mathcal{C}_{IJ}^I), \max_I \max_J \mathcal{D}_{IJ}^R + \mathcal{L}(\max_I \max_J \mathcal{D}_{IJ}^I), \min_I \min_J \mu_{IJ}^R + \mathcal{L}(\min_I \min_J \mu_{IJ}^I), \min_I \min_J \eta_{IJ}^R + \mathcal{L}(\min_I \min_J \eta_{IJ}^I) \right)$ ,  $I, J = 1, 2, \dots, \tilde{\alpha}$ , thus  $\mathbb{A}_{BC}^- \leq BCIFSWG(\mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}\alpha}) \leq \mathbb{A}_{BC}^+$ . (Boundedness)

Proof: Let  $\mathbb{A}_{BC}^- = \left( \min_I \min_J \mathcal{C}_{IJ}^R + \mathcal{L}(\min_I \min_J \mathcal{C}_{IJ}^I), \min_I \min_J \mathcal{D}_{IJ}^R + \mathcal{L}(\min_I \min_J \mathcal{D}_{IJ}^I), \max_I \max_J \mu_{IJ}^R + \mathcal{L}(\max_I \max_J \mu_{IJ}^I), \max_I \max_J \eta_{IJ}^R + \mathcal{L}(\max_I \max_J \eta_{IJ}^I) \right)$  and  $\mathbb{A}_{BC}^+ =$

$$\left( \max_I \max_J \mathcal{C}_{IJ}^R + L \left( \max_I \max_J \mathcal{C}_{IJ}^I \right), \max_I \max_J \mathcal{D}_{IJ}^R + L \left( \max_I \max_J \mathcal{D}_{IJ}^I \right), \min_I \min_J \mu_{IJ}^R + L \left( \min_I \min_J \mu_{IJ}^I \right), \min_I \min_J \eta_{IJ}^R + L \left( \min_I \min_J \eta_{IJ}^I \right) \right), I, J = 1, 2, \dots, \tilde{\alpha}_A, \text{ Thus}$$

$$BCIFSWG \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}_A} \right) \leq BCIFSWG \left( \mathbb{A}_{BC}^{+11}, \mathbb{A}_{BC}^{+12}, \dots, \mathbb{A}_{BC}^{+\tilde{\alpha}_A} \right) = \mathbb{A}_{BC}^+$$

$$BCIFSWG \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}_A} \right) \geq BCIFSWG \left( \mathbb{A}_{BC}^{-11}, \mathbb{A}_{BC}^{-12}, \dots, \mathbb{A}_{BC}^{-\tilde{\alpha}_A} \right) = \mathbb{A}_{BC}^-$$

Thus

$$\mathbb{A}_{BC}^- \leq BCIFSWG \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}_A} \right) \leq \mathbb{A}_{BC}^+$$

Definition 7: Consider any family of BCIFSNs  $\mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}_A}$ . Thus, the technique of BCIFSOWG operator based on  $\mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}_A}$  is mentioned and developed by:

$$BCIFSOWG \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}_A} \right) = \otimes_{J=1}^{\tilde{\alpha}_A} \left( \otimes_{I=1}^{\tilde{\alpha}_A} \left( \mathbb{A}_{BC}^{o(I)o(J)} \right)^{z_{\psi}^I} \right)^{z_{\Omega}^J}$$

Notice that  $z_{\Omega}^J, z_{\psi}^I \in [0, 1]$  represent weight vectors with the condition of  $\sum_{I=1}^{\tilde{\alpha}_A} z_{\psi}^I = 1$  and  $\sum_{J=1}^{\tilde{\alpha}_A} z_{\Omega}^J = 1$ , with order  $o(I), o(J) \leq o(I - 1), o(J - 1)$ .

Theorem 4: Prove that the aggregated value of the  $BCIFSWG \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}_A} \right)$  is als a BCIFSN, such as

$$BCIFSWG \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}\mathfrak{A}} \right)$$

$$= \left( \begin{array}{l} \left( \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}} \left( \mathbb{C}_{o(I)o(J)}^R \right)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}} \left( \mathbb{C}_{o(I)o(J)}^I \right)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( - \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}} \left( |\mathbb{D}_{o(I)o(J)}^R| \right)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + L \left( - \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}} \left( |\mathbb{D}_{o(I)o(J)}^I| \right)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( 1 - \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}} \left( 1 - \mathfrak{r}_{o(I)o(J)}^R \right)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \\ L \left( 1 - \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}} \left( 1 - \mathfrak{r}_{o(I)o(J)}^I \right)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right), \\ \left( -1 + \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}} \left( |1 + \mathfrak{h}_{o(I)o(J)}^R| \right)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right) + \\ L \left( -1 + \prod_{J=1}^{\mathfrak{A}} \left( \prod_{I=1}^{\tilde{\alpha}} \left( |1 + \mathfrak{h}_{o(I)o(J)}^I| \right)^{z_{\psi}^I} \right)^{z_{\Omega}^J} \right). \end{array} \right)$$

Property 10: When  $\mathbb{A}_{BC}^{IJ} = \mathbb{A}_{BC} = (\mathbb{C}^R + L\mathbb{C}^I, \mathbb{D}^R + L\mathbb{D}^I, \mathfrak{r}^R + L\mathfrak{r}^I, \mathfrak{h}^R + L\mathfrak{h}^I)$ ,  $I, J = 1, 2, \dots, \tilde{\alpha}\mathfrak{A}$ , thus  $BCIFSOWG \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}\mathfrak{A}} \right) = \mathbb{A}_{BC}$ . (Idempotency)

Property 11: When  $\mathbb{A}_{BC}^{IJ} \leq \mathbb{A}_{BC}^{*IJ}$ , thus  $BCIFSOWG \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}\mathfrak{A}} \right) \leq BCIFSOWG \left( \mathbb{A}_{BC}^{*11}, \mathbb{A}_{BC}^{*12}, \dots, \mathbb{A}_{BC}^{*\tilde{\alpha}\mathfrak{A}} \right)$ . (Monotonicity)

Property 12: When  $\mathbb{A}_{BC}^- = \left( \min_I \min_J \mathbb{C}_{IJ}^R + L \left( \min_I \min_J \mathbb{C}_{IJ}^I \right), \min_I \min_J \mathbb{D}_{IJ}^R + L \left( \min_I \min_J \mathbb{D}_{IJ}^I \right), \max_I \max_J \mathfrak{r}_{IJ}^R + L \left( \max_I \max_J \mathfrak{r}_{IJ}^I \right), \max_I \max_J \mathfrak{h}_{IJ}^R + L \left( \max_I \max_J \mathfrak{h}_{IJ}^I \right) \right)$ , and  $\mathbb{A}_{BC}^+ = \left( \max_I \max_J \mathbb{C}_{IJ}^R + L \left( \max_I \max_J \mathbb{C}_{IJ}^I \right), \max_I \max_J \mathbb{D}_{IJ}^R + L \left( \max_I \max_J \mathbb{D}_{IJ}^I \right), \min_I \min_J \mathfrak{r}_{IJ}^R + L \left( \min_I \min_J \mathfrak{r}_{IJ}^I \right), \min_I \min_J \mathfrak{h}_{IJ}^R + L \left( \min_I \min_J \mathfrak{h}_{IJ}^I \right) \right)$ ,  $I, J = 1, 2, \dots, \tilde{\alpha}\mathfrak{A}$ , thus  $\mathbb{A}_{BC}^- \leq BCIFSOWG \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\tilde{\alpha}\mathfrak{A}} \right) \leq \mathbb{A}_{BC}^+$ . (Boundedness)

#### 4. Multi-Attribute Decision-Making Techniques

In this section, we calculate the procedure of the decision-making technique or multi-attribute decision-making technique based on designed operators, called BCIFSWA operator and BCIFSWG operator to improve the reliability and effectiveness of the invented models.

Let  $A_{BC}^1, A_{BC}^2, \dots, A_{BC}^{\alpha}$  be a family of alternatives with a collection of attributes  $A_A^1, A_A^2, \dots, A_A^{\tilde{\alpha}}$ . Further, we also consider the finite values of parameters  $\tilde{\alpha}_I, I = 1, 2, \dots, n$  for the collection of alternatives. The duplet  $(A_{BCIFS}, \mathbb{E})$  represents a BCIFSS, such as  $A_{BCIFS2}: \mathbb{E} \rightarrow BCIFS(\mathbb{Y})$ . The function in CIFSS is denoted and defined by:  $C^R, C^I, \mu^R, \mu^I: \mathbb{Y} \rightarrow [0, 1]$  and  $\mathcal{D}^R, \mathcal{D}^I, \eta^R, \eta^I: \mathbb{Y} \rightarrow [-1, 0]$  with  $C^R(\tilde{\alpha}), C^I(\tilde{\alpha}) \in [0, 1]$ ,  $\mu^R(\tilde{\alpha}), \mu^I(\tilde{\alpha}) \in [0, 1]$  and  $\mathcal{D}^R(\tilde{\alpha}), \mathcal{D}^I(\tilde{\alpha}) \in [-1, 0]$ ,  $\eta^R(\tilde{\alpha}), \eta^I(\tilde{\alpha}) \in [-1, 0]$ . Finally, the BCIFSN is mentioned by:  $A_{BC}^{IJ} = (C_{IJ}^R + LC_{IJ}^I, \mathcal{D}_{IJ}^R + L\mathcal{D}_{IJ}^I, \mu_{IJ}^R + L\mu_{IJ}^I, \eta_{IJ}^R + L\eta_{IJ}^I), I, J = 1, 2, \dots, \tilde{\alpha}, \alpha$ . Thus, we goal to construct the decision matrix by including the values of BCIFSN for evaluating the decision-making procedure, therefore, the major steps of the decision-making technique are described by:

Step 1: Compute the matrix of BCIFSN: For this, we arrange the collection of alternatives and for each alternative, we consider the collection of attributes, then we define a matrix, such as

$$M = \begin{bmatrix} A_{BC}^{11} & A_{BC}^{12} & \dots & A_{BC}^{1\alpha} \\ A_{BC}^{21} & A_{BC}^{22} & \dots & A_{BC}^{2\alpha} \\ \vdots & \vdots & \ddots & \vdots \\ A_{BC}^{\tilde{\alpha}1} & A_{BC}^{\tilde{\alpha}2} & \dots & A_{BC}^{\tilde{\alpha}\alpha} \end{bmatrix}$$

Step 2: Compute the matrix of normalization: For this, we goal to assess the arranged data in the decision matrix for the evaluation of normalization in the case of cost types of information, such as

$$N = \begin{cases} (C_{IJ}^R + LC_{IJ}^I, \mathcal{D}_{IJ}^R + L\mathcal{D}_{IJ}^I, \mu_{IJ}^R + L\mu_{IJ}^I, \eta_{IJ}^R + L\eta_{IJ}^I) & \text{for benefit} \\ (\mu_{IJ}^R + L\mu_{IJ}^I, \eta_{IJ}^R + L\eta_{IJ}^I, C_{IJ}^R + LC_{IJ}^I, \mathcal{D}_{IJ}^R + L\mathcal{D}_{IJ}^I) & \text{for cost} \end{cases}$$

But in the case of benefit type of information, we do not go for normalization.

Step 3: Design the aggregated matrix: We goal to evaluate the aggregation of normalization of the matrix based on the BCIFSWA operator and BCIFSWG operator, such as

$$BCIFSWA \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\check{\alpha}^{\mathfrak{r}}}\right)$$

$$= \left( \begin{array}{l} \left( 1 - \prod_{J=1}^{\mathfrak{r}} \left( \prod_{I=1}^{\check{\alpha}} (1 - \mathfrak{C}_{IJ}^R)^{z_{\Omega}^I} \right)^{z_{\Omega}^J} \right) + \\ \mathbb{L} \left( 1 - \prod_{J=1}^{\mathfrak{r}} \left( \prod_{I=1}^{\check{\alpha}} (1 - \mathfrak{C}_{IJ}^L)^{z_{\Omega}^I} \right)^{z_{\Omega}^J} \right), \\ \left( -1 + \prod_{J=1}^{\mathfrak{r}} \left( \prod_{I=1}^{\check{\alpha}} (1 + \mathfrak{D}_{IJ}^R)^{z_{\Omega}^I} \right)^{z_{\Omega}^J} \right) + \\ \mathbb{L} \left( -1 + \prod_{J=1}^{\mathfrak{r}} \left( \prod_{I=1}^{\check{\alpha}} (1 + \mathfrak{D}_{IJ}^L)^{z_{\Omega}^I} \right)^{z_{\Omega}^J} \right), \\ \left( \prod_{J=1}^{\mathfrak{r}} \left( \prod_{I=1}^{\check{\alpha}} (\mathfrak{M}_{IJ}^R)^{z_{\Omega}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( \prod_{J=1}^{\mathfrak{r}} \left( \prod_{I=1}^{\check{\alpha}} (\mathfrak{M}_{IJ}^L)^{z_{\Omega}^I} \right)^{z_{\Omega}^J} \right), \\ \left( -\prod_{J=1}^{\mathfrak{r}} \left( \prod_{I=1}^{\check{\alpha}} (\mathfrak{N}_{IJ}^R)^{z_{\Omega}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( -\prod_{J=1}^{\mathfrak{r}} \left( \prod_{I=1}^{\check{\alpha}} (\mathfrak{N}_{IJ}^L)^{z_{\Omega}^I} \right)^{z_{\Omega}^J} \right). \end{array} \right)$$

and

$$BCIFSWG \left( \mathbb{A}_{BC}^{11}, \mathbb{A}_{BC}^{12}, \dots, \mathbb{A}_{BC}^{\check{\alpha}^{\mathfrak{r}}}\right)$$

$$= \left( \begin{array}{l} \left( \prod_{J=1}^{\mathfrak{r}} \left( \prod_{I=1}^{\check{\alpha}} (\mathfrak{C}_{IJ}^R)^{z_{\Omega}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( \prod_{J=1}^{\mathfrak{r}} \left( \prod_{I=1}^{\check{\alpha}} (\mathfrak{C}_{IJ}^L)^{z_{\Omega}^I} \right)^{z_{\Omega}^J} \right), \\ \left( -\prod_{J=1}^{\mathfrak{r}} \left( \prod_{I=1}^{\check{\alpha}} (\mathfrak{D}_{IJ}^R)^{z_{\Omega}^I} \right)^{z_{\Omega}^J} \right) + \mathbb{L} \left( -\prod_{J=1}^{\mathfrak{r}} \left( \prod_{I=1}^{\check{\alpha}} (\mathfrak{D}_{IJ}^L)^{z_{\Omega}^I} \right)^{z_{\Omega}^J} \right), \\ \left( 1 - \prod_{J=1}^{\mathfrak{r}} \left( \prod_{I=1}^{\check{\alpha}} (1 - \mathfrak{M}_{IJ}^R)^{z_{\Omega}^I} \right)^{z_{\Omega}^J} \right) + \\ \mathbb{L} \left( 1 - \prod_{J=1}^{\mathfrak{r}} \left( \prod_{I=1}^{\check{\alpha}} (1 - \mathfrak{M}_{IJ}^L)^{z_{\Omega}^I} \right)^{z_{\Omega}^J} \right), \\ \left( -1 + \prod_{J=1}^{\mathfrak{r}} \left( \prod_{I=1}^{\check{\alpha}} (1 + \mathfrak{N}_{IJ}^R)^{z_{\Omega}^I} \right)^{z_{\Omega}^J} \right) + \\ \mathbb{L} \left( -1 + \prod_{J=1}^{\mathfrak{r}} \left( \prod_{I=1}^{\check{\alpha}} (1 + \mathfrak{N}_{IJ}^L)^{z_{\Omega}^I} \right)^{z_{\Omega}^J} \right). \end{array} \right)$$

Step 4: Analyze the Score values of the aggregated theory based on the Score function, such as

$$\mathcal{ZC}(\mathbb{A}_{BC}^{IJ}) = \frac{1}{8} \left( (\mathcal{C}_{IJ}^R + \mathcal{C}_{IJ}^I + \mathcal{D}_{IJ}^R + \mathcal{D}_{IJ}^I) - (\mu_{IJ}^R + \mu_{IJ}^I + \eta_{IJ}^R + \eta_{IJ}^I) \right) \in [-1, 1]$$

$$AC(\mathbb{A}_{BC}^{IJ}) = \frac{1}{8} \left( (\mathcal{C}_{IJ}^R + \mathcal{C}_{IJ}^I - \mathcal{D}_{IJ}^R - \mathcal{D}_{IJ}^I) + (\mu_{IJ}^R + \mu_{IJ}^I - \eta_{IJ}^R - \eta_{IJ}^I) \right) \in [0, 1]$$

Where, the following information is also very important for the above theory, such as: When  $\mathcal{ZC}(\mathbb{A}_{BC}^{11}) > \mathcal{ZC}(\mathbb{A}_{BC}^{12})$ , thus  $\mathbb{A}_{BC}^{11} > \mathbb{A}_{BC}^{12}$ , When  $AC(\mathbb{A}_{BC}^{11}) > AC(\mathbb{A}_{BC}^{12})$ , thus  $\mathbb{A}_{BC}^{11} > \mathbb{A}_{BC}^{12}$ .

Step 5: Finally, we analyze the ranking values of the aggregated values for the evaluation of the best preference among the collection of preferences.

## 5. Enhancing Cybersecurity Risk Assessment Techniques: A Comprehensive Approaches

Cybersecurity threats are a great risk nowadays for every organization in this digital fast-paced world. To stay ahead in the business these types of threats whether in the form of data breaches, ransomware or advanced digital threats are making it difficult for organizations. As of these continuously changing threats sophistication and increased frequency organizations need their risk assessment models to become more robust, adaptable, and comprehensive to counter the data breaches. To eliminate modern complexities of cybersecurity threats traditional approaches to cybersecurity risk assessment are no longer useful. There is a great need for organizations to invest improve and enhancing their cybersecurity risk assessment models by incorporating new technologies, methodologies, and advanced strategies to effectively eliminate these risks. This article describes the cybersecurity risk assessment model's accuracy, timely response, and actions against potential threats by providing the key areas in which the cybersecurity risk assessment model can be improved and enhanced. For organizations to become more effective in risk elimination a more resilient, precise, and proactive approach must be adopted by implementing a collaboration framework amongst departments, continuous risk monitoring, real-time intelligence, and machine learning AI systems. For a deep understanding of cybersecurity incidents and strengthening risk management tactics dynamic risk models and business impact analysis must be adopted. This article outlines cyber security risks modernization and assessments with the key improvements in detail.

- 1) Data Integration and Real-time Threat Intelligence: Threat intelligence in real-time is crucial in improving the risk assessment model of cyber security. Knowing the emerging cyber-attack methods and up-to-date knowledge about cyber threats is crucial. To allow organizations to have a proactive approach toward potential risks the threat intelligence platforms provide insights about the vectors of current attacks and vulnerabilities. Businesses can greatly improve in aspects of cyber security by integrating key data about the threats in their risk assessment models accordingly. Moreover, there is a need for organizations to increase their source of data to create a more comprehensive risk assessment model. Within the organization's infrastructure internet data, security incidents, and alerts provide valuable information about the ongoing threats and vulnerabilities. A comprehensive picture of

potential threats can be gained by a more holistic way of data collection. The risk assessment model should also include external data, a public threat database, and exposures and common vulnerability systems.

- 2) **Artificial Intelligence and Machine Learning:** To enhance the effectiveness of an organization's risk assessment models towards sophisticated cyber threats Artificial intelligence and machine learning technologies provide very helpful insights and solutions rapidly. The great number of datasets and patterns of attack history can be quickly analyzed through machine learning algorithms. Organizations can effectively improve their real-time detection of threats without involving human intervention by using AI-based models, that can detect unusual traffic patterns, anomalies in the system, and network and security breaches while real-time monitoring. Machine learning technology also provides useful predictive analytics for cyber-attacks. Machine learning models can predict future cyber-attacks based on the previous history of the risks and vulnerabilities in the organization's system. This predictive analysis of machine learning models allows organizations to focus more on their resources for the elimination of the potentially high risks before they even emerge materialistically. Therefore, reducing the damage caused by potential cyber-attacks.
- 3) **Prioritization and Advanced Risk Scoring:** To categorize risks the traditional method of risk assessment models uses low, medium, and high-risk types. The simplicity of this approach often fails to understand the potential impact and sophistication of the specific risks. For the organizations to improve their risk assessment models towards potential threats risk scoring system which is more quantitative and granular should be adopted. With this system likelihood of occurrence of risk is measured as well as the cyber attack's operational, reputational, and financial impact on the organization. To mitigate the risks more effectively the organizations should adopt a more refined risk scoring model. To ensure most critical vulnerabilities are addressed first the risks should be quantified based on impact and severity, thus allocating the organizational resources strategically.
- 4) **Continuous Vulnerability Management:** Continuous vulnerability management is another important enhancement in the risk assessment model. Periodic-based threat assessment is no longer adequate with the continuously changing threat nature. As new vulnerabilities may arise at times, organizations must implement continuous threat monitoring and scanning. Businesses can monitor vulnerabilities in real-time with the help of automated risk assessment models and can take immediate actions to eliminate new threats. Cyberattack exposure to any organization can be significantly reduced by timely patching with the updated risk assessment data. A vulnerability cannot always be an immediate risk but as the vulnerability is exposed over time it can become an easy target for the attackers. The system with patched and updated security updates significantly reduces the attacker's opportunity window to harm the organization's crucial data. As of the current trend of attacks, organizations can determine which vulnerability poses the greatest threat with the help of risk models which can also assess the vulnerability's exploitations.
- 5) **Business Impact Analysis (BIA):** Understanding of potential risk impact of a cyberattack on business the improvement in the risk assessment model is the key aspect. Moreover, where identification of vulnerabilities is important another key aspect is the organization's

assessment of the effects caused by these risks on the reputation, finances, and operations of the organization. There is a system analysis called Business impact analysis which helps organizations to assess the effects of the potential cyberattacks such as ransomware, data breaches, and disruption of services. To mitigate the effects of cyberattacks decision makers can prioritize which risks are more critical according to the complete understanding of the impact scope. Organizations can allocate their resources according to the decisions based on the business impact analysis data. Cost balancing is also crucial for the cybersecurity mitigation measures caused by the security breaches on the operational and financial losses.

Moreover, to find the best and worst one among the above five, we aim to use the following four attributes such as growth analysis, social impact, environmental impact, and political impact with four parameters:  $\ddot{a}_I, I = 1, 2, 3, 4$ . The weight vectors for the above attributes and parameter values are explained in the form, such as:  $(0.3, 0.3, 0.2, 0.2)^T$  and  $(0.2, 0.3, 0.4, 0.1)^T$ . Thus, we goal to construct the decision matrix by including the values of BCIFSN for evaluating the decision-making procedure, therefore, the major steps of the decision-making technique are described by:

Step 1: Compute the matrix of BCIFSN: For this, we arrange the collection of alternatives and for each alternative, we consider the collection of attributes, and then we define a matrix, see Table 1 to Table 5.

**Table 1.** Decision matrix for alternative  $A_A^1$ .

	$\ddot{a}_1$	$\ddot{a}_2$	$\ddot{a}_3$	$\ddot{a}_4$
$A_{BC}^1$	$\left( \begin{array}{l} (0.5, 0.4), \\ (-0.6, -0.3), \\ (0.3, 0.1), \\ (-0.3, -0.1) \end{array} \right)'$	$\left( \begin{array}{l} (0.51, 0.41), \\ (-0.61, -0.31), \\ (0.31, 0.11), \\ (-0.31, -0.11) \end{array} \right)'$	$\left( \begin{array}{l} (0.52, 0.42), \\ (-0.62, -0.32), \\ (0.32, 0.12), \\ (-0.32, -0.12) \end{array} \right)'$	$\left( \begin{array}{l} (0.53, 0.43), \\ (-0.63, -0.33), \\ (0.33, 0.13), \\ (-0.33, -0.13) \end{array} \right)'$
$A_{BC}^2$	$\left( \begin{array}{l} (0.4, 0.3), \\ (-0.3, -0.2), \\ (0.2, 0.4), \\ (-0.2, -0.2) \end{array} \right)'$	$\left( \begin{array}{l} (0.41, 0.31), \\ (-0.31, -0.21), \\ (0.21, 0.41), \\ (-0.21, -0.21) \end{array} \right)'$	$\left( \begin{array}{l} (0.42, 0.32), \\ (-0.32, -0.22), \\ (0.22, 0.42), \\ (-0.22, -0.22) \end{array} \right)'$	$\left( \begin{array}{l} (0.43, 0.33), \\ (-0.33, -0.23), \\ (0.23, 0.43), \\ (-0.23, -0.23) \end{array} \right)'$
$A_{BC}^3$	$\left( \begin{array}{l} (0.3, 0.2), \\ (-0.2, -0.3), \\ (0.1, 0.3), \\ (-0.3, -0.1) \end{array} \right)'$	$\left( \begin{array}{l} (0.31, 0.21), \\ (-0.21, -0.31), \\ (0.11, 0.31), \\ (-0.31, -0.11) \end{array} \right)'$	$\left( \begin{array}{l} (0.32, 0.22), \\ (-0.22, -0.32), \\ (0.12, 0.32), \\ (-0.32, -0.12) \end{array} \right)'$	$\left( \begin{array}{l} (0.33, 0.23), \\ (-0.23, -0.33), \\ (0.13, 0.33), \\ (-0.33, -0.13) \end{array} \right)'$
$A_{BC}^4$	$\left( \begin{array}{l} (0.2, 0.1), \\ (-0.1, -0.1), \\ (0.2, 0.2), \\ (-0.1, -0.2) \end{array} \right)'$	$\left( \begin{array}{l} (0.21, 0.11), \\ (-0.11, -0.11), \\ (0.21, 0.21), \\ (-0.11, -0.21) \end{array} \right)'$	$\left( \begin{array}{l} (0.22, 0.12), \\ (-0.12, -0.12), \\ (0.22, 0.22), \\ (-0.12, -0.22) \end{array} \right)'$	$\left( \begin{array}{l} (0.23, 0.13), \\ (-0.13, -0.13), \\ (0.23, 0.23), \\ (-0.13, -0.23) \end{array} \right)'$

**Table 2.** Decision matrix for alternative  $A_A^2$ .

	$\ddot{a}_1$	$\ddot{a}_2$	$\ddot{a}_3$	$\ddot{a}_4$
$A_{BC}^1$	$\left( \begin{array}{l} (0.6, 0.3), \\ (-0.4, -0.2), \\ (0.1, 0.1), \\ (-0.2, -0.4) \end{array} \right)'$	$\left( \begin{array}{l} (0.61, 0.31), \\ (-0.41, -0.21), \\ (0.11, 0.11), \\ (-0.21, -0.41) \end{array} \right)'$	$\left( \begin{array}{l} (0.62, 0.32), \\ (-0.42, -0.22), \\ (0.12, 0.12), \\ (-0.22, -0.42) \end{array} \right)'$	$\left( \begin{array}{l} (0.63, 0.33), \\ (-0.43, -0.23), \\ (0.13, 0.13), \\ (-0.23, -0.43) \end{array} \right)'$



**Table 5.** Decision matrix for alternative  $A_A^5$ .

	$\ddot{a}_1$	$\ddot{a}_2$	$\ddot{a}_3$	$\ddot{a}_4$
$A_{BC}^1$	$\left( \begin{matrix} (0.2, 0.3), \\ (-0.3, -0.1) \end{matrix} \right)'$	$\left( \begin{matrix} (0.21, 0.31), \\ (-0.31, -0.11) \end{matrix} \right)'$	$\left( \begin{matrix} (0.22, 0.32), \\ (-0.32, -0.12) \end{matrix} \right)'$	$\left( \begin{matrix} (0.23, 0.33), \\ (-0.33, -0.13) \end{matrix} \right)'$
$A_{BC}^2$	$\left( \begin{matrix} (0.3, 0.3), \\ (-0.2, -0.2) \end{matrix} \right)'$	$\left( \begin{matrix} (0.31, 0.31), \\ (-0.21, -0.21) \end{matrix} \right)'$	$\left( \begin{matrix} (0.32, 0.32), \\ (-0.22, -0.22) \end{matrix} \right)'$	$\left( \begin{matrix} (0.33, 0.33), \\ (-0.23, -0.23) \end{matrix} \right)'$
$A_{BC}^3$	$\left( \begin{matrix} (0.4, 0.4), \\ (-0.3, -0.2) \end{matrix} \right)'$	$\left( \begin{matrix} (0.41, 0.41), \\ (-0.31, -0.21) \end{matrix} \right)'$	$\left( \begin{matrix} (0.42, 0.42), \\ (-0.32, -0.22) \end{matrix} \right)'$	$\left( \begin{matrix} (0.43, 0.43), \\ (-0.33, -0.23) \end{matrix} \right)'$
$A_{BC}^4$	$\left( \begin{matrix} (0.5, 0.5), \\ (-0.1, -0.2) \end{matrix} \right)'$	$\left( \begin{matrix} (0.51, 0.51), \\ (-0.11, -0.21) \end{matrix} \right)'$	$\left( \begin{matrix} (0.52, 0.52), \\ (-0.12, -0.22) \end{matrix} \right)'$	$\left( \begin{matrix} (0.53, 0.53), \\ (-0.13, -0.23) \end{matrix} \right)'$

Step 2: Compute the matrix of normalization: For this, we goal to assess the arranged data in the decision matrix for the evaluation of normalization in the case of cost types of information, such as

$$N = \begin{cases} (C_{IJ}^R + LC_{IJ}^I, \mathfrak{D}_{IJ}^R + L\mathfrak{D}_{IJ}^I, \mu_{IJ}^R + L\mu_{IJ}^I, \eta_{IJ}^R + L\eta_{IJ}^I) & \text{for benefit} \\ (\mu_{IJ}^R + L\mu_{IJ}^I, \eta_{IJ}^R + L\eta_{IJ}^I, C_{IJ}^R + LC_{IJ}^I, \mathfrak{D}_{IJ}^R + L\mathfrak{D}_{IJ}^I) & \text{for cost} \end{cases}$$

But in the case of benefit type of information, we do not go for normalization. Anyhow the data in Table 1 to Table 5 is ok.

Step 3: Design the aggregated matrix: We goal to evaluate the aggregation of normalization of the matrix based on the BCIFSWA operator and BCIFSWG operator, see Table 6.

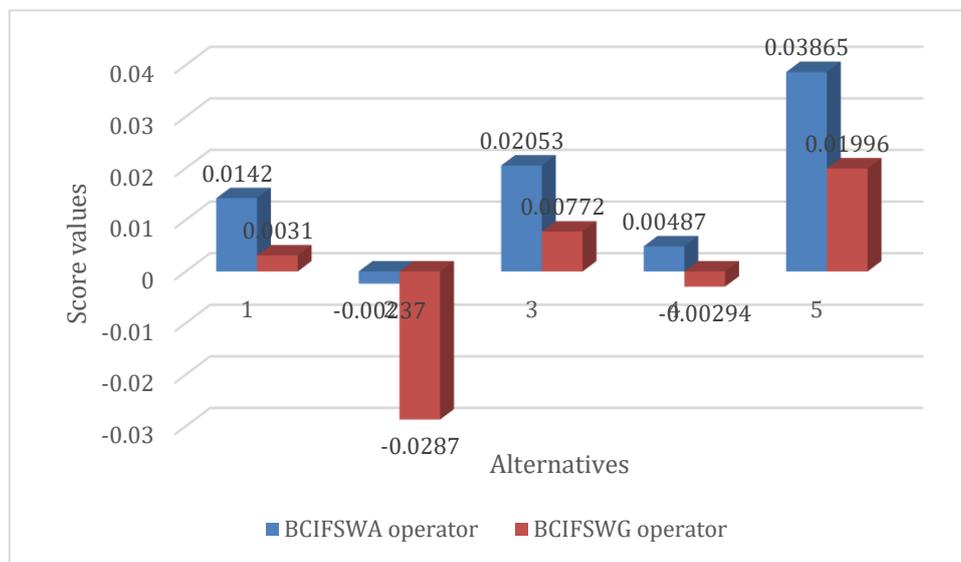
**Table 6.** Decision matrix based on aggregation operators.

	BCIFSWA operator	BCIFSWG operator
$A_A^1$	$\left( \begin{matrix} ((0.3799, 0.2789), (-0.3366, -0.2659)), \\ ((0.1873, 0.2665), (-0.2515, -0.1451)) \end{matrix} \right)$	$\left( \begin{matrix} ((0.3611, 0.2558), (-0.2768, -0.2515)), \\ ((0.1966, 0.3009), (-0.3023, -0.2589)) \end{matrix} \right)$
$A_A^2$	$\left( \begin{matrix} ((0.3798, 0.1869), (-0.2875, -0.1645)), \\ ((0.2338, 0.1972), (-0.2921, -0.2726)) \end{matrix} \right)$	$\left( \begin{matrix} ((0.2652, 0.1670), (-0.2726, -0.1546)), \\ ((0.2274, 0.2169), (-0.3743, -0.3046)) \end{matrix} \right)$
$A_A^3$	$\left( \begin{matrix} ((0.5131, 0.4131), (-0.5131, -0.4131)), \\ ((0.4230, 0.1923), (-0.1995, -0.2515)) \end{matrix} \right)$	$\left( \begin{matrix} ((0.5128, 0.4128), (-0.5128, -0.4128)), \\ ((0.4131, 0.2181), (-0.2671, -0.3023)) \end{matrix} \right)$
$A_A^4$	$\left( \begin{matrix} ((0.2366, 0.2445), (-0.2445, -0.2845)), \\ ((0.3611, 0.2558), (-0.2515, -0.2786)) \end{matrix} \right)$	$\left( \begin{matrix} ((0.2185, 0.2388), (-0.2388, -0.2786)), \\ ((0.3799, 0.2789), (-0.3023, -0.3199)) \end{matrix} \right)$
$A_A^5$	$\left( \begin{matrix} ((0.3596, 0.3768), (-0.2659, -0.1940)), \\ ((0.1873, 0.2420), (-0.2515, -0.1451)) \end{matrix} \right)$	$\left( \begin{matrix} ((0.3399, 0.3672), (-0.2515, -0.1872)), \\ ((0.1966, 0.2559), (-0.3023, -0.2589)) \end{matrix} \right)$

Step 4: Analyze the Score values of the aggregated theory based on the Score function, see Table 7, where the geometrical interpretation of the data in Table 7 is listed in Figure 2.

**Table 7.** Decision matrix based on Score values.

	BCIFSWA operator	BCIFSWG operator
$A_A^1$	0.0142	0.0031
$A_A^2$	-0.00237	-0.0287
$A_A^3$	0.02053	0.00772
$A_A^4$	0.00487	-0.00294
$A_A^5$	0.03865	0.01996



**Figure 2.** Graphical abstract of data in Table 7.

Step 5: Finally, we analyze the ranking values of the aggregated values for the evaluation of the best preference among the collection of preferences, see Table 8.

**Table 8.** Ranking values information.

Methods	Ranking values	Best one
BCIFSWA operator	$A_A^5 > A_A^3 > A_A^1 > A_A^4 > A_A^2$	$A_A^5$
BCIFSWG operator	$A_A^5 > A_A^3 > A_A^1 > A_A^4 > A_A^2$	$A_A^5$

The best preference is  $A_A^5$  according to a tool of the proposed BCIFSWA operator and BCIFSWG operator, which represented the Business Impact Analysis (BIA): Understanding of potential risk impact of a cyberattack on business the improvement in the risk assessment model is the key aspect. Moreover, where identification of vulnerabilities is important another key aspect is the organization's assessment of the effects caused by these risks on the reputation, finances, and operations of the organization. There is a system analysis called Business impact analysis which helps organizations to assess the effects of the potential cyberattacks such as ransomware, data breaches, and disruption

of services. To mitigate the effects of cyberattacks decision makers can prioritize which risks are more critical according to the complete understanding of the impact scope. Organizations can allocate their resources according to the decisions based on the business impact analysis data. Cost balancing is also crucial for the cybersecurity mitigation measures caused by the security breaches on the operational and financial losses. Further, we discuss the comparative analysis of the proposed and existing models to show the supremacy of the designed operators.

## 6. Comparative Analysis

In this section, we compare the ranking tool of the proposed models with the ranking tool of the existing information to evaluate the sovereignty and rationality of the designed approaches. For this, we aim to arrange some prevailing tools based on fuzzy information and their related techniques. Therefore, we consider the following existing tools, such as Xu [24] developed the aggregation operators for IFSs. Xu and Yager [25] proposed the intuitionistic fuzzy geometric operators. Arora and Garg [26] invented the intuitionistic fuzzy soft robust operators. Jana et al. [27] designed the bipolar fuzzy soft robust operators. Ali and Yang [28] proposed the bipolar complex fuzzy soft robust operators. Gwak et al. [29] explored the hybrid structure based on bipolar complex fuzzy soft sets. Riaz and Tehrim [30] described the cubic bipolar fuzzy geometric operators. Therefore, using the data in Table 1 to Table 5, we illustrate the ranking values in Table 9.

**Table 9.** Representation of the comparative analysis.

Methods	Score values	Ranking values
Xu [24]	0.0,0.0,0.0,0.0,0.0	Failed
Xu and Yager [25]	0.0,0.0,0.0,0.0,0.0	Failed
Arora and Garg [26]	0.0,0.0,0.0,0.0,0.0	Failed
Jana et al. [27]	0.0,0.0,0.0,0.0,0.0	Failed
Ali and Yang [28]	0.0,0.0,0.0,0.0,0.0	Failed
Gwak et al. [29]	0.0,0.0,0.0,0.0,0.0	Failed
Riaz and Tehrim [30]	0.0,0.0,0.0,0.0,0.0	Failed
BCIFSWA operator	0.0142,-0.00237,0.02053,0.00487,0.03865	$A_A^5 > A_A^3 > A_A^1 > A_A^4 > A_A^2$
BCIFSWG operator	0.0031,-0.0287,0.00772,-0.00294,0.01996	$A_A^5 > A_A^3 > A_A^1 > A_A^4 > A_A^2$

The best preference is  $A_A^5$  according to a tool of the proposed BCIFSWA operator and BCIFSWG operator, which represented the Business Impact Analysis (BIA): Understanding of potential risk impact of a cyberattack on business the improvement in the risk assessment model is the key aspect. Further, the prevailing techniques have failed because of their limitations and problems, and due to this reason, they failed to cope with the data in Table 1 to Table 5. Hence, the proposed theory is more reliable and more efficient than existing models.

## 7. Conclusions

The major contribution of this manuscript is described below:

- 1) We describe the cybersecurity risk assessment model's accuracy, timely response, and actions against potential threats by providing the key areas in which the cybersecurity risk assessment model can be improved and enhanced.
- 2) We propose the model of robust aggregation operators based on BCIFS information, called the BCIFSWA operator, BCIFSOWA operator, BCIFSWG operators, and BCIFSOWG operators, with some practical properties, called idempotency, monotonicity, and boundedness.
- 3) We discuss the MADM model based on proposed operators for evaluating the enhancement of the cybersecurity risk assessment techniques with a comprehensive approach.
- 4) We illustrate numerical examples based on decision-making techniques for assessing the comparative analysis among the proposed ranking values with existing ranking values to show the worth and verity of the designed models.

In the future, we will enhance the theory of robust operators for bipolar Pythagorean fuzzy information with some new extensions, such as Z-number extensions [31] and LMAW and DNMA Techniques [32]. Then, we will discuss their application in clustering analysis, medical diagnosis, artificial intelligence, and data science to improve the worth of the proposed theory.

### Author Contributions

The following statements should be used "Conceptualization, Z.A. and A.R.; methodology, Z.A.; software, A.R.; validation, Z.A., A.R. and K.M.; formal analysis, Z.A.; investigation, A.R.; resources, Z.A.; data curation, Z.A.; writing—original draft preparation, Z.A.; writing—review and editing, Z.A.; visualization, Z.A.; supervision, Z.A.; project administration, Z.A.; funding acquisition, Z.A. All authors have read and agreed to the published version of the manuscript." Authorship must be limited to those who have contributed substantially to the work reported.

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### Data Availability Statement

The data utilized in this manuscript are hypothetical and artificial, and one can use these data before prior permission by just citing this manuscript.

### Conflicts of Interest

About the publication of this manuscript, the authors declare that they have no conflict of interest.

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### References

- [1] Li, Y., & Liu, Q. (2021). A comprehensive review study of cyber-attacks and cyber security; Emerging trends and recent developments. *Energy Reports*, 7, 8176-8186. <https://doi.org/10.1016/j.egy.2021.08.126>
- [2] Snider, K. L., Shandler, R., Zandani, S., & Canetti, D. (2021). Cyberattacks, cyber threats, and attitudes toward cybersecurity policies. *Journal of Cybersecurity*, 7(1), tyab019. <https://doi.org/10.1093/cybsec/tyab019>

- [3] Minnaar, A. (2014). 'Crackers', cyberattacks and cybersecurity vulnerabilities: the difficulties in combatting the new cybercriminals. *Acta Criminologica: African Journal of Criminology & Victimology*, 27(sed-2), 127-144. <https://hdl.handle.net/10520/EJC171548>
- [4] Chudasama, D., & Dungarani, R. (2022). A Comparative Study About Cyberattacks and Cybersecurity in Real World. *Journal of Network Security*, 10(3), 15-23p. DOI (Journal): 10.37591/JoNS
- [5] Norris, D. F., Mateczun, L., Joshi, A., & Finin, T. (2019). Cyberattacks at the grassroots: American local governments and the need for high levels of cybersecurity. *Public Administration Review*, 79(6), 895-904. <https://doi.org/10.1111/puar.13028>
- [6] Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, 8(3), 338-353. DOI: <https://doi.org/10.2307/2272014>
- [7] Atanassov, K. Intuitionistic fuzzy sets. In VII ITKR's Session; Deposited in Central Sci.—Techn. Library of Bulg. Acad. of Sci., 1697/84; Sofia, Bulgaria, June 1983. (In Bulgarian) [https://doi.org/10.1007/978-3-7908-1870-3\\_1](https://doi.org/10.1007/978-3-7908-1870-3_1)
- [8] Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20(1), 87-96. [https://doi.org/10.1007/978-3-7908-1870-3\\_2](https://doi.org/10.1007/978-3-7908-1870-3_2)
- [9] Xiong, F., Siddique, N., Ali, Z., & Yin, S. (2024). Conditional aggregation operator defined by the power information concerning type-2 fuzzy deep learning algorithm for financial investment data decision-making. *IEEE Access*, 12, 96672-96690. DOI: 10.1109/ACCESS.2024.3425056
- [10] Ali, Z., & Hila, K. (2025). Disturbance Observer-Based Control: Weighted Aggregated Aczel-Alsina Sum Product Assessment Based on Power Operators for Managing Fuzzy 2-Tuple Linguistic Neural Networks. *Cognitive Computation*, 17(1), 1-18. <https://doi.org/10.1007/s12559-024-10371-4>
- [11] Ramot, D., Milo, R., Friedman, M., & Kandel, A. (2002). Complex fuzzy sets. *IEEE Transactions on Fuzzy Systems*, 10(2), 171-186. DOI: 10.1109/91.995119
- [12] Alkouri, A. M. D. J. S., & Salleh, A. R. (2012). Complex intuitionistic fuzzy sets. In *AIP conference proceedings. American Institute of Physics*. 1482(1), 464-470. <https://doi.org/10.1063/1.4757515>
- [13] Mahmood, T., Idrees, A., Hayat, K., & Ashiq, M. (2024). Selection of AI Architecture for Autonomous Vehicles Using Complex Intuitionistic Fuzzy Rough Decision Making. *World Electric Vehicle Journal*, 15(9). **10.3390/wevj15090402**
- [14] Bisht, D., Singh, Y., Goyal, G., & Kumar, R. (2024). Intuitionistic fuzzy set entropy: A novel measure for uncertainty management in complex group decision-making. *Mathematics in Engineering, Science & Aerospace (MESA)*, 15(2). <https://nonlinearstudies.com/index.php/ mesa/article/view/3635>
- [15] Ali, Z., Emam, W., Mahmood, T., & Wang, H. (2024). Archimedean Heronian mean operators based on complex intuitionistic fuzzy sets and their applications in decision-making problems. *Heliyon*, 10(3). <https://doi.org/10.1016/j.heliyon.2024.e24767>
- [16] Zhang, W. R. (1998, May). (Yin)(Yang) bipolar fuzzy sets. In *1998 IEEE International Conference on Fuzzy Systems proceedings. IEEE World Congress on computational intelligence (Cat. No. 98CH36228)* (Vol. 1, pp. 835-840). IEEE. DOI: 10.1109/FUZZY.1998.687599
- [17] Mahmood, T., & Ur Rehman, U. (2022). A novel approach towards bipolar complex fuzzy sets and their applications in generalized similarity measures. *International Journal of Intelligent Systems*, 37(1), 535-567. <https://doi.org/10.1002/int.22639>
- [18] Lee, K. M., LEE, K. M., & CIOS, K. J. (2001). Comparison of interval-valued fuzzy sets, intuitionistic fuzzy sets, and bipolar-valued fuzzy sets. In *Computing and information technologies: exploring emerging technologies* (pp. 433-439). [https://doi.org/10.1142/9789812810885\\_0055](https://doi.org/10.1142/9789812810885_0055)
- [19] Molodtsov, D. (1999). Soft set theory—first results. *Computers & mathematics with applications*, 37(4-5), 19-31. [https://doi.org/10.1016/S0898-1221\(99\)00056-5](https://doi.org/10.1016/S0898-1221(99)00056-5)
- [20] Abdullah, S., Aslam, M., & Ullah, K. (2014). Bipolar fuzzy soft sets and their applications in decision-making problems. *Journal of Intelligent & Fuzzy Systems*, 27(2), 729-742. DOI: 10.3233/IFS-131031
- [21] Alqaraleh, S. M., Abd Ulazeez, M. J. S., Massa'deh, M. O., Talafha, A. G., & Bataihah, A. (2022). Bipolar complex fuzzy soft sets and their application. *International Journal of Fuzzy System Applications (IJFSA)*, 11(1), 1-23. DOI: 10.4018/IJFSA.285551
- [22] Jana, C., & Pal, M. (2018). Application of bipolar intuitionistic fuzzy soft sets in decision-making problems. *International Journal of Fuzzy System Applications (IJFSA)*, 7(3), 32-55. DOI: 10.4018/IJFSA.2018070103
- [23] Ali, Z. (2024). Analysis of BCK/BCI-Algebras Based on Bipolar Complex Intuitionistic Fuzzy Soft Ideals. *Neutrosophic Systems with Applications*, 24, 34-50. <https://doi.org/10.61356/j.nswa.2024.24419>
- [24] Xu, Z. (2007). Intuitionistic fuzzy aggregation operators. *IEEE Transactions on fuzzy systems*, 15(6), 1179-1187. DOI: 10.1109/TFUZZ.2006.890678
- [25] Xu, Z., & Yager, R. R. (2006). Some geometric aggregation operators are based on intuitionistic fuzzy sets. *International journal of general systems*, 35(4), 417-433. <https://doi.org/10.1080/03081070600574353>

- [26] Arora, R., & Garg, H. (2018). Robust aggregation operators for multi-criteria decision-making with intuitionistic fuzzy soft set environment. *Scientia Iranica*, 25(2), 931-942. doi: 10.24200/sci.2017.4433
- [27] Jana, C., Pal, M., & Wang, J. (2019). A robust aggregation operator for multi-criteria decision-making method with bipolar fuzzy soft environment. *Iranian Journal of Fuzzy Systems*, 16(6), 1-16. [https://ijfs.usb.ac.ir/article\\_5014\\_ce85118040f486b4363f37fc17de39f7.pdf](https://ijfs.usb.ac.ir/article_5014_ce85118040f486b4363f37fc17de39f7.pdf)
- [28] Ali, Z., & Yang, M. S. (2024). Improving Risk Assessment Model for Cyber Security Using Robust Aggregation Operators for Bipolar Complex Fuzzy Soft Inference Systems. *Mathematics*, 12(4), 582. <https://doi.org/10.3390/math12040582>
- [29] Gwak, J., Garg, H., & Jan, N. (2023). Hybrid integrated decision-making algorithm for clustering analysis based on bipolar complex fuzzy and soft sets. *Alexandria Engineering Journal*, 67, 473-487. <https://doi.org/10.1016/j.aej.2022.12.003>
- [30] Riaz, M., & Tehrim, S. T. (2020). Cubic bipolar fuzzy set with application to multi-criteria group decision-making using geometric aggregation operators. *Soft Computing*, 24(21), 16111-16133. <https://doi.org/10.1007/s00500-020-04927-3>
- [31] Moslem, S. (2025). Evaluating Commuters' Travel Mode Choice Using the Z-Number Extension of Parsimonious Best Worst Method. *Applied Soft Computing*, 112918. <https://doi.org/10.1016/j.asoc.2025.112918>
- [32] Moslem, S., Gholamizadehc, K., Zarei, E., Pasman, H. J., Martinez-Pastor, B., & Pilla, F. (2025). A Comparative Assessment of Domino Accident Analysis Methods in Process Industries Using LMAW and DNMA Techniques. *Reliability Engineering & System Safety*, 110981. <https://doi.org/10.1016/j.res.2025.110981>