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Designing and Analyzing Fermatean Fuzzy Decision Models: A Comprehensive R Programming Approach

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ABSTRACT

This paper investigates the integration of fuzzy set theory with decision-making methodologies in the context of Multi-Criteria Decision Making (MCDM). It provides a foundational overview of fuzzy set theory and essential concepts for understanding decision-making processes. The study implements two methodologies—Logarithmic Percentage Change Driven Objective Weighting (LOPCOW) for criteria weight determination and the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) for alternative ranking—using R programming. By integrating these methodologies, the study enhances decision-making accuracy by objectively deriving weights and systematically ranking alternatives. Through a detailed numerical illustration, the study demonstrates the computational efficiency and reliability of this approach in solving complex MCDM problems. The findings underscore the significance of combining LOPCOW with TOPSIS to improve the robustness and transparency of decision analysis across various practical applications.

1. Introduction

The advancement of decision-making frameworks is pivotal in addressing complex real-world problems, where the intricacies of multiple criteria must be considered simultaneously. The concept of fuzzy sets, introduced by Zadeh (1965) [1], has revolutionized decision-making by accommodating uncertainty and vagueness in the evaluation process. Over the years, fuzzy set theory has evolved, with variants such as Fermatean fuzzy sets gaining prominence due to their ability to represent higher degrees of uncertainty and hesitancy [17].

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These sets extend the applicability of decision-making frameworks to domains requiring nuanced judgments, such as environmental sustainability, engineering design, and financial assessment.

In recent times, the integration of Fermatean fuzzy sets with Multi-Criteria Decision-Making (MCDM) methodologies has emerged as a powerful tool for resolving decision-making challenges [13]. MCDM approaches like TOPSIS and LOPCOW have been widely utilized due to their robustness and computational simplicity. TOPSIS, for instance, excels in ranking alternatives based on their proximity to ideal solutions [7], while LOPCOW is instrumental in deriving objective weights for criteria [11]. Both methods, when augmented by fuzzy set theory, offer unparalleled precision in handling intricate decision scenarios.

The use of computational tools such as R programming further enhances the effectiveness of these methodologies. As a versatile and powerful statistical computing environment, R facilitates the implementation of sophisticated algorithms, making it a preferred choice for decision analysts and researchers. By leveraging R, the computational challenges associated with MCDM models—such as handling large datasets, performing matrix operations, and visualizing results—are efficiently addressed [16]. This integration not only improves accuracy but also enables researchers to adapt these methodologies to diverse application domains.

In particular, the integration of Fermatean fuzzy sets with LOPCOW and TOPSIS methodologies represents a significant leap forward in decision-making research. Fermatean fuzzy sets offer a richer representation of uncertainty compared to their predecessors, such as intuitionistic and Pythagorean fuzzy sets [4]. This enhanced capability is particularly beneficial in scenarios where decision criteria are inherently imprecise, as seen in environmental assessments, supplier selection, and material evaluation [18].

Moreover, the dynamic nature of the Logarithmic Percentage Change Driven Objective Weighting (LOPCOW) method allows for the capture of subtle variations in criteria, ensuring that significant factors are adequately prioritized. When paired with TOPSIS, this approach not only identifies the most suitable alternatives but also provides a clear rationale for the decision-making process, enhancing transparency and reliability [6].

This paper aims to explore the integration of these advanced methodologies within the Fermatean fuzzy decision-making paradigm, utilizing R programming for implementation and analysis. A case study involving the selection of optimal alternatives underscores the practical applicability and robustness of this framework. By presenting a comprehensive analysis, this study seeks to advance the understanding of Fermatean fuzzy decision-making while demonstrating its potential for solving complex, multidimensional problems.

The rest of the paper is structured as follows: Section 2 reviews relevant literature on fuzzy sets and MCDM methods. Section 3 outlines the preliminaries of Fermatean fuzzy sets. Section 4 presents the proposed methodology, detailing the algorithms for LOPCOW and TOPSIS. Sections 5 and 6 applies the proposed approach to a real-world case study, followed by a discussion of the results. Finally, Section 7 concludes the study and offers directions for future research.

2. Literature Review

The advancement of Multi-Criteria Decision-Making (MCDM) methodologies has been a subject of extensive research, resulting in significant progress in theory and application across various domains. This section provides an overview of key studies that have contributed to the development and application of MCDM and fuzzy set theories.

Sahoo and Goswami (2023) presented a comprehensive review of MCDM methods, highlighting recent advancements and their applications in decision-making processes. They emphasized the importance of integrating computational techniques to enhance the adaptability and robustness of MCDM frameworks in solving complex real-world problems [19].

Alsalem et al. (2022) examined the use of MCDM in managing challenges associated with the COVID-19 pandemic. Their review underscored the utility of MCDM for prioritizing resource allocation and strategic planning during health crises. They provided a theoretical analysis of various approaches and emphasized the need for incorporating uncertainty-handling mechanisms like fuzzy logic [2].

Chen and Pan (2021) explored the role of fuzzy MCDM in construction management, focusing on the use of network-based approaches. Their study demonstrated the effectiveness of fuzzy methods in addressing vagueness and ambiguity inherent in construction decision-making processes [8].

Kahraman and Gündoğdu (2021) introduced decision-making frameworks utilizing spherical fuzzy sets. Their work advanced fuzzy set theory by enabling more comprehensive representation of uncertainty, which is critical in scenarios requiring nuanced decision criteria [3].

Yu et al. (2022) provided an evolutionary perspective on intuitionistic fuzzy set theory. They analyzed the dynamic progression of this theory and its applications, emphasizing its significance in fields like decision sciences and engineering [20].

Dymova et al. (2021) proposed a user-friendly computational implementation of fuzzy MCDM approaches. Their work highlighted the role of advanced computational tools in improving the accessibility and efficiency of decision-making models [10].

Castelló-Sirvent (2022) utilized fuzzy-set qualitative comparative analysis to evaluate the body of literature on fuzzy sets. His research provided valuable insights into the trends and gaps in fuzzy set theory research [9].

Gul (2018) reviewed the application of fuzzy MCDM in occupational health and safety risk assessments. He identified the advantages of fuzzy methodologies in capturing uncertainty and variability in risk evaluation [12].

Zare et al. (2016) conducted a systematic review of MCDM approaches in e-learning environments. They classified various methodologies and highlighted the increasing relevance of MCDM in educational technology [21].

Akram et al. (2023) enhanced the ELECTRE II method using 2-tuple linguistic m-polar fuzzy sets for group decision-making. Their work showcased the flexibility of fuzzy approaches in handling linguistic data and subjective evaluations [4].

Baumann et al. (2019) reviewed MCDM methods for evaluating energy storage systems. Their study provided a critical assessment of the methodologies employed and underscored the importance of incorporating sustainability metrics in decision-making [5].

Liao et al. (2018) reviewed the hesitant fuzzy linguistic term set and its applications in decision-making. Their work demonstrated the potential of hesitant fuzzy logic in scenarios involving conflicting or incomplete information [15].

These studies collectively demonstrate the versatility and effectiveness of MCDM methods, particularly when combined with fuzzy set theories, in addressing complex decision-making challenges. The integration of advanced computational tools and the development of novel fuzzy frameworks, such as intuitionistic, Pythagorean, and Fermatean fuzzy sets, have significantly enhanced the capability of MCDM methodologies to handle uncertainty and vagueness. These advancements are particularly evident in applications spanning healthcare, construction management, energy systems, and education.

As decision-making scenarios grow increasingly complex, the use of hybrid models combining different MCDM approaches and fuzzy theories is gaining prominence. Specifically, we emphasize how the integration of LOPCOW with TOPSIS under a Fermatean fuzzy environment offers improved decision robustness and accuracy.

3. Definitions and Concepts

3.1 Fuzzy Set

Definition 1 Let U be a universal set. A fuzzy set (FS) A in U is defined as:

$$A = \{(u, \mu_A(u)) : u \in U\},$$

where $\mu_A(u) : U \to [0,1]$ is called the membership function, and $\mu_A(u)$ represents the degree of membership of an element $u \in U$.

3.2 Intuitionistic Fuzzy Set

Defenition 2 An intuitionistic fuzzy set (IFS) on a non-empty set U is defined as:

$$\mathscr{I} = \{\langle u, \alpha_{\mathscr{I}}(u), \beta_{\mathscr{I}}(u)\rangle : u \in U\},$$

where $\alpha_{\mathscr{I}}(u) : U \to [0,1]$ and $\beta_{\mathscr{I}}(u) : U \to [0,1]$ denote the membership degree and non-membership degree, respectively, of each $u \in U$. The condition $0 \leq \alpha_{\mathscr{I}}(u) + \beta_{\mathscr{I}}(u) \leq 1$ must hold for all $u \in U$. When $\beta_{\mathscr{I}}(u) = 1 - \alpha_{\mathscr{I}}(u)$ for all $u \in U$, the set $\mathscr{I}$ reduces to a fuzzy set.

3.3 Pythagorean Fuzzy Set

Definition 3 A Pythagorean fuzzy set (PFS) on a non-empty set U is defined as:

$$\mathscr{P} = \{\langle u, \alpha_{\mathscr{P}}(u), \beta_{\mathscr{P}}(u)\rangle : u \in U\},$$

where $\alpha_{\mathscr{P}}(u) : U \to [0,1]$ and $\beta_{\mathscr{P}}(u) : U \to [0,1]$ denote the membership and non-membership degrees, respectively, of $u \in U$. The condition $0 \leq (\alpha_{\mathscr{P}}(u))^2 + (\beta_{\mathscr{P}}(u))^2 \leq 1$ must hold for all $u \in U$. For each $u \in U$, the degree of indeterminacy is given by:

$$\gamma_{\mathscr{P}}(u) = \sqrt{1 - (\alpha_{\mathscr{P}}(u))^2 - (\beta_{\mathscr{P}}(u))^2}.$$

3.4 Fermatean Fuzzy Set

Definition 4 Let U be a universe of discourse. A Fermatean fuzzy set (FFS) $\mathscr{F}$ in U is defined as:

$$\mathscr{F} = \{\langle u, \alpha_{\mathscr{F}}(u), \beta_{\mathscr{F}}(u)\rangle : u \in U\},$$

where $\alpha_{\mathscr{F}}(u) : U \to [0,1]$ and $\beta_{\mathscr{F}}(u) : U \to [0,1]$ denote the membership and non-membership degrees, respectively. The condition $0 \leq (\alpha_{\mathscr{F}}(u))^3 + (\beta_{\mathscr{F}}(u))^3 \leq 1$ must hold for all $u \in U$. For each $u \in U$, the degree of indeterminacy is given by:

$$\gamma_{\mathscr{F}}(u) = \sqrt[3]{1 - (\alpha_{\mathscr{F}}(u))^3 - (\beta_{\mathscr{F}}(u))^3}.$$

For simplicity, the Fermatean fuzzy set $\mathscr{F}$ can be denoted as $\mathscr{F} = (\alpha_{\mathscr{F}}, \beta_{\mathscr{F}})$.

3.5 Fermatean Fuzzy Set Score Function

Definition 5 Let $\mathscr{F} = (\alpha_{\mathscr{F}}, \beta_{\mathscr{F}})$ be a Fermatean fuzzy set. The score function of $\mathscr{F}$ is defined as:

$$S(\mathscr{F}) = (\alpha_{\mathscr{F}})^3 - (\beta_{\mathscr{F}})^3.$$

4. Proposed Methodology

4.1 Algorithm of LOPCOW Method

Logarithmic Percentage Change Driven Objective Weighting (LOPCOW) is a method used in MCDM to determine the weights of different criteria. This approach involves calculating the logarithmic percentage change of each criterion across different alternatives. The weights are then assigned based on these changes, with more significant changes being assigned higher weights. This method aims to emphasize criteria that exhibit more significant variations among alternatives, thus capturing their relative importance in the decision-making process. The algorithm of the LOPCOW method is presented as follows:

Step 1: Normalization of the decision-matrix.

LOPCOW applies the linear max-min type of normalization approach to normalize the initial decision matrix. Accordingly, the elements of the normalized decision matrix are obtained as $\mathscr{R} = [r_{ij}]_{m\times n}$ where,

$r_{ij} = \dfrac{x_{ij} - x^j_{min}}{x^j_{max} - x^j_{min}}$ (when $j \in j^+$, effect direction: maximize)

$r_{ij} = \dfrac{x^j_{max} - x_{ij}}{x^j_{max} - x^j_{min}}$ (when $j \in j^-$, effect direction: minimize)

Step 2: Obtain the Percentage Value (PV) for the criteria.

The PV for each criterion is calculated as

$$P_j = \left| ln\left(\frac{\sqrt{\frac{\sum\limits_{i=a}^{m} r_{ij}^2}{m}}}{\sigma}\right).100\right|$$

σ is the standard deviation of the performance values of the alternatives under a specific criterion.

Step 3: Deduce the criteria weights.

The weights for the j^{th} criterion is calculated by using

$w_j = \dfrac{P_j}{\sum\limits_{j=1}^{n} P_j}$ where, ($\sum\limits_{j=1}^{n} w_j = 1$).

4.2 Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS)

TOPSIS is a MCDM method used to evaluate and rank alternatives based on multiple criteria. The algorithm of the TOPSIS method is presented as follows:

Step 1: Construct the normalized decision matrix R

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum\limits_{i=1}^{m} x_{ij}^2}} \tag{1}$$

Step 2: Construct weighted normalized decision matrix

$$v_{ij} = w_j r_{ij} \tag{2}$$

Here, $\sum\limits_{j=1}^{n} w_j = 1, w_j$ is the weight of j th criterion.
Step 3: Determine the positive-ideal solution (PIS) and negative ideal solution (NIS), denoted respectively as A^+ and A^-, are defined in the following way.

$$\begin{aligned} A^+ &= \{(max\, v_{ij} | j \in J)\, or\, (min\, v_{ij} | j \in J')\}, i = 1, 2, \cdots, m & (3)\\ &= \{v_1^+, v_2^+, \cdots, v_n^+\} & (4)\\ A^- &= \{(min\, v_{ij} | j \in J)\, or\, (max\, v_{ij} | j \in J')\}, i = 1, 2, \cdots, m & (5)\\ &= \{v_1^-, v_2^-, \cdots, v_n^-\} & (6) \end{aligned}$$

where J and J' are sets of benefit and cost criteria, respectively.
Step 4: Calculate the distances of each alternative from PIS and NIS

$$S_i^+ = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^+\right)^2}, for i = 1, 2, \ldots, m \tag{7}$$

$$S_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^-\right)^2}, for i = 1, 2, \ldots, m \tag{8}$$

Step 5: Calculate the closeness coefficient and rank the order of alternatives

$$C_i^+ = \frac{S_i^-}{S_i^+ + S_i^-}, 0 < C_i^+ < 1, i = 1, 2, \ldots, m \tag{9}$$

where $C_i^+ \in [0, 1]$ with $i = 1, 2, \ldots, m$. The best alternative can therefore be found according to the preference order of C_i^+. The value is the more the better. If C_i^+ is close to 1 , it indicate the alternative A_i is closer to the PIS.

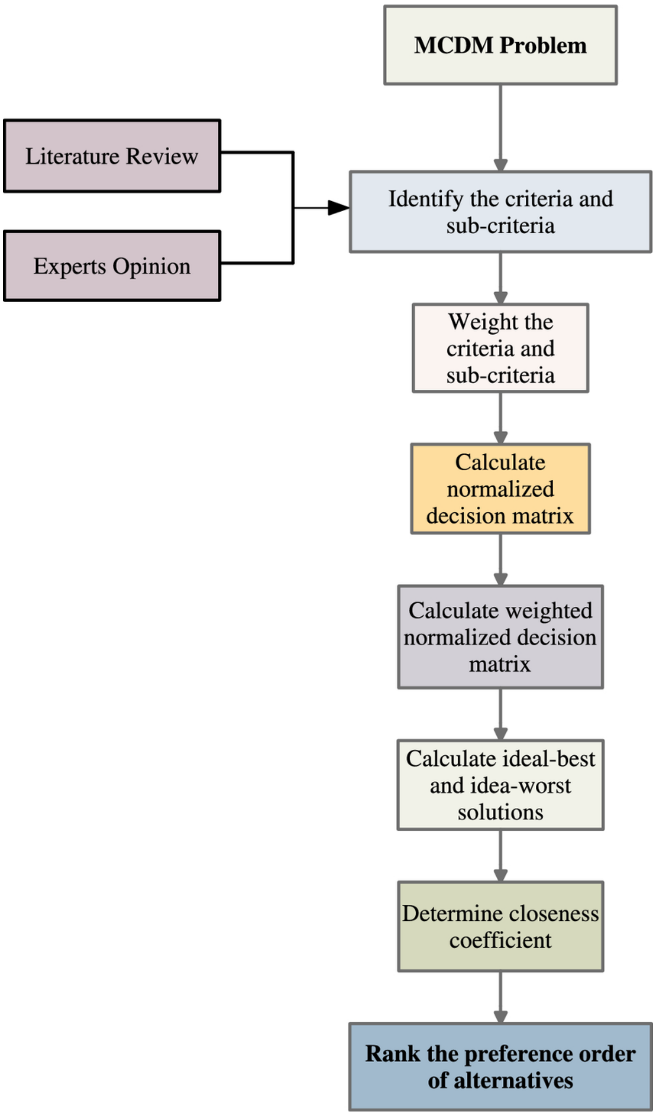

Figure 1: Flowchart depicting the proposed methodology steps

5. Case Study

In this case study, we apply the Fermatean fuzzy set approach to select the best video camera among five alternatives, denoted as A_i $(i = 1, 2, 3, 4, 5)$. The goal is to evaluate these five cameras, identify the most appropriate option, and rank them based on their relative importance.

5.1 *Alternatives*

The five different types of cameras chosen for evaluation are described as follows:

- **Digital Single-Lens Reflex (DSLR) Camera (A_1)**
  A DSLR camera features a fixed digital sensor, autofocus capabilities, and the ability to store thousands of photos on its internal memory card. It is a popular choice for both professional and entry-level photographers.
- **Mirrorless Camera (A_2)**
  A mirrorless camera operates on a principle similar to that of a DSLR, but without a reflex mirror.

Unlike a DSLR, which uses a mirror to direct light to the viewfinder and image sensor, a mirrorless camera does not rely on this mechanism.

- **Compact Camera (A_3)**
  Also known as a point-and-shoot camera, a compact camera is designed for simplicity and portability. It is typically smaller and lighter than DSLRs or mirrorless cameras, making it convenient for casual use.

- **Bridge Camera (A_4)**
  A bridge camera serves as an intermediate option between point-and-shoot cameras and DSLRs. It offers some manual controls, a long-range zoom lens, and a viewfinder, though its lenses are usually non-interchangeable.

- **Action Camera (A_5)**
  An action camera, or action cam, is built for recording dynamic activities. These cameras are compact, rugged, and often waterproof, featuring CMOS image sensors and support for burst mode, time-lapse photography, and high-definition video recording.

5.2 *Criteria*

To determine the best video camera, six beneficial criteria are considered:

- **Resolution (C_1)**
  Resolution refers to the level of detail in an image, measured by the number of pixels. Higher resolution provides more detail and definition in captured images.

- **Cost (C_2)**
  Cost encompasses the monetary price and any associated resources or sacrifices required to acquire the camera.

- **Lens Quality (C_3)**
  Lens quality evaluates the optical performance of a camera lens, including factors such as sharpness, clarity, contrast, color reproduction, and control over aberrations.

- **Frames Per Second (FPS) (C_4)**
  FPS measures the number of frames displayed or recorded per second in a video sequence, indicating the frame rate of the camera.

- **Overall Operational Speed (C_5)**
  This criterion assesses the camera's efficiency in performing tasks such as focusing, shutter response, startup time, continuous shooting, and menu navigation.

- **Autofocus Speed (C_6)**
  Autofocus speed measures how quickly and accurately the camera's autofocus system adjusts to achieve sharp and clear images, crucial for capturing moving subjects or spontaneous moments.

6. Evaluation of the numerical study

Step 1: The initial decision-making matrix was created. Here, five alternatives and six criteria have been considered. Based on the linguistic scale provided in Table 3.1, the performance value of the alternatives and criteria was established.

Step 2: The FFSs linguistic decision-making matrix, as shown in Table 3.2, was formulated.
Step 3: Using the score function defined in Definition 3.5, the FFS score matrix was calculated, which is shown in Table 3.3.

Table 1: Linguistic scale for FFS

Linguistic terms	FFN
Very Low	(0.1, 0.75)
Low	(0.25, 0.6)
Medium	(0.4,0.5)
High	(0.7, 0.2)
Very High	(0.8, 0.1)

Table 2: Decision matrix with linguistic terms

	$\mathscr{C}_1$	$\mathscr{C}_2$	$\mathscr{C}_3$	$\mathscr{C}_4$	$\mathscr{C}_5$	$\mathscr{C}_6$
$\mathscr{A}_1$	(0.7,0.2)	(0.1,0.75)	(0.7,0.2)	(0.25,06)	(0.25,0.6)	(0.4, 0.5)
$\mathscr{A}_2$	(0.8, 0.1)	(0.25,0.6)	(0.8, 0.1)	(0.4, 0.5)	(0.4, 0.5)	(0.7, 0.2)
$\mathscr{A}_3$	(0.25,0.6)	(0.7, 0.2)	(0.25,0.6)	(0.1,0.75)	(0.1,0.75)	(0.1,0.75)
$\mathscr{A}_4$	(0.4, 0.5)	(0.4, 0.5)	(0.4, 0.5)	(0.7, 0.2)	(0.7, 0.2)	(0.8, 0.1)
$\mathscr{A}_5$	(0.1,0.75)	(0.8, 0.1)	(0.1,0.75)	(0.8, 0.1)	(0.8, 0.1)	(0.25,0.6)

Table 3: FFS score matrix

	$\mathscr{C}_1$	$\mathscr{C}_2$	$\mathscr{C}_3$	$\mathscr{C}_4$	$\mathscr{C}_5$	$\mathscr{C}_6$
$\mathscr{A}_1$	0.335	-0.421	0.335	-0.200	-0.200	-0.061
$\mathscr{A}_2$	0.511	-0.200	0.511	-0.061	-0.061	0.335
$\mathscr{A}_3$	-0.200	0.335	-0.200	-0.421	-0.421	-0.421
$\mathscr{A}_4$	-0.061	-0.061	-0.061	0.335	0.335	0.511
$\mathscr{A}_5$	-0.421	0.511	0.421	0.511	0.511	-0.200

6.1 Weight evaluation by LOPCOW R-Programming

The following is the calculation of the criterion weights by the LOPCOW method using the R programming language as shown in Figure 2 and Table 4.

```
# Function to calculate LOPCOW weights
lopcow <- function(matrix_data) {
# Calculate logarithmic percentage changes
log_changes <- apply(matrix_data, 2, function(col) {
diff(log(col + 1))  # Add 1 to avoid log(0)
})

# Calculate absolute sum of changes for each criterion
abs_sum_changes <- colSums(abs(log_changes))

# Normalize the sum of changes to get weights
weights <- abs_sum_changes / sum(abs_sum_changes)

return(weights)
}

# Example matrix (5x8) with decimal values
matrix_data <- matrix(c(0.335,-0.421,0.335,-0.200,-0.200,0.061,
0.511,-0.200,0.511,-0.061,-0.061,0.335,
-0.200,0.335,-0.200,-0.421,-0.421,-0.421,
-0.061,-0.061,-0.061,0.335,0.335,0.511,
-0.421,0.511,0.421,0.511,0.511,-0.200),
nrow = 5, byrow = TRUE)

# Calculate LOPCOW weights
weights <- lopcow(matrix_data)

# Print the weights
print(weights)
[1] 0.1366997 0.1619748 0.1299584 0.1561282 0.1561282 0.2591106
```

Table 4: Weights of the criteria

$\mathscr{C}_j$	$\mathscr{C}_1$	$\mathscr{C}_2$	$\mathscr{C}_3$	$\mathscr{C}_4$	$\mathscr{C}_5$	$\mathscr{C}_6$
w_j	0.1366997	0.1619748	0.1299584	0.1561282	0.1561282	0.2591106

6.2 Ranking the alternatives by TOPSIS R-Programming

The following is the ranking of the alternatives as determined by the TOPSIS technique with the R programming language.

```
\begin{lstlisting}[language=R]
# Function to calculate TOPSIS scores using LOPCOW weights
topsis_with_lopcow_weights <- function(matrix_data, weights) {
```

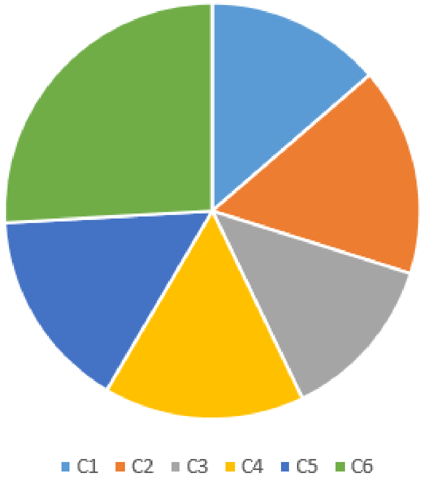

Figure 2: Criteria weights

```
# Normalize the matrix
normalized_matrix <- t(apply(matrix_data, 1, function(row)
row / sqrt(sum(row^2))))

# Calculate weighted normalized matrix
weighted_normalized_matrix <- normalized_matrix * weights

# Determine the ideal and anti-ideal solutions
ideal_solution <- apply(weighted_normalized_matrix, 2, max)
anti_ideal_solution <- apply(weighted_normalized_matrix, 2, min)

# Calculate the Euclidean distances to the ideal and anti-ideal
solutions
distance_to_ideal <- sqrt(rowSums((weighted_normalized_matrix -
ideal_solution)^2))
distance_to_anti_ideal <- sqrt(rowSums
((weighted_normalized_matrix - anti_ideal_solution)^2))

# Calculate the TOPSIS scores
topsis_score <- distance_to_anti_ideal / (distance_to_ideal +
distance_to_anti_ideal)

# Return the TOPSIS scores
return(topsis_score)
}

# Example matrix (5x8) with decimal values
matrix_data <- matrix(c(0.335,-0.421,0.335,-0.200,-0.200,0.061,
0.511,-0.200,0.511,-0.061,-0.061,0.335,
-0.200,0.335,-0.200,-0.421,-0.421,-0.421,
-0.061,-0.061,-0.061,0.335,0.335,0.511,
-0.421,0.511,0.421,0.511,0.511,-0.200),
nrow = 5, byrow = TRUE)
```

```
# LOPCOW weights (you should replace this with your own weights)
lopcow_weights <-
c(0.1366997,0.1619748,0.1299584,0.1561282,0.1561282,0.2591106)

# Calculate TOPSIS scores using LOPCOW weights
topsis_scores <- topsis_with_lopcow_weights(matrix_data,
lopcow_weights)

# Print the TOPSIS scores
print(topsis_scores)
[1] 0.4387238 0.6149259 0.3575969 0.6219212 0.5601593
```

Table 5: Ranking value

Alternatives	Utility value	Rank
A_1	0.4387238	4
A_2	0.6149259	2
A_3	0.3575969	5
A_4	0.6219212	1
A_5	0.5601593	3

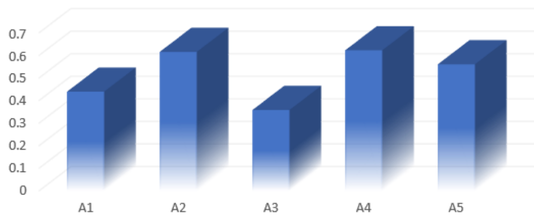

Figure 3: Ranking value of alternatives

The alternative A_4 has the highest utility value and it is considered as the best choice among the alternatives as shown in Figure 3 and Table 5.

7. Results and Discussion

The study utilized the Fermatean fuzzy set approach to evaluate and rank five video camera alternatives based on six beneficial criteria: resolution, cost, lens quality, frames per second, operational speed, and autofocus speed. Employing R programming, the Logarithmic Percentage Change Driven Objective Weighting (LOPCOW) method was implemented to objectively evaluate criterion weights, which emphasized significant variations in performance. These weights were then integrated into the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method to derive the final rankings.

The results from LOPCOW indicated that autofocus speed $(\mathscr{C}_6)$ held the highest weight (0.259), reflecting its critical role in determining camera performance. Other criteria, such as cost $(\mathscr{C}_2)$ and frames per second ($\mathscr{C}_4$), also demonstrated significant importance with weights of 0.161 and 0.156, respectively. This weighting process underscores the ability of LOPCOW to capture the nuances of decision-making scenarios with varied criteria.

Using these weights, the TOPSIS rankings revealed that the Bridge Camera ($\mathscr{A}_4$) emerged as the top-performing alternative, achieving the highest closeness coefficient (0.621). The Mirrorless Camera ($\mathscr{A}_2$) secured the second position (0.615), while the Action Camera ($\mathscr{A}_5$) followed closely in third place (0.560). The Digital Single-Lens Reflex (DSLR) Camera ($\mathscr{A}_1$) ranked fourth (0.439), and the Compact Camera ($\mathscr{A}_3$) was deemed the least favorable option (0.358). These rankings align with the practical considerations of professional and recreational photography, validating the robustness of the integrated framework.

Specifically, R provides robust libraries for fuzzy set computations and MCDM techniques, enabling efficient processing of complex mathematical operations. Its vectorized computations and parallel processing capabilities enhance speed and scalability, making it well-suited for high-dimensional decision-making problems. Additionally, compared to traditional spreadsheet-based approaches, R significantly reduces computational time and improves result accuracy, ensuring the reliability of the proposed LOPCOW-TOPSIS framework in real-world applications.

8. Conclusion

This paper demonstrates an effective method for solving the Fermatean LOPCOW-TOPSIS problem using the R programming language, presenting a robust framework for multi-criteria decision-making. By integrating the Logarithmic Percentage Change Driven Objective Weighting (LOPCOW) method with the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), it highlights the capability to derive objective weights from datasets containing decimal values. These weights, reflecting the relative importance of criteria, enhance the accuracy of alternative assessments. Applying the TOPSIS algorithm with LOPCOW weights enables ranking alternatives based on their closeness to ideal and anti-ideal solutions, providing valuable insights for decision-making. Specifically, we emphasize the practical utility of the proposed LOPCOW-TOPSIS framework in addressing complex multi-criteria decision-making problems under uncertainty. The study demonstrates its effectiveness in real-world applications by providing a more reliable ranking mechanism, reducing subjectivity in weight assignments, and ensuring robust decision-making in renewable energy selection. Additionally, we discuss its applicability to broader domains such as supply chain optimization and infrastructure project evaluation.

R programming has been a critical tool in this research, offering extensive data analysis capabilities and a rich ecosystem of packages for statistical computations and visualizations. The flexibility of R allows for efficient data preprocessing, calculation of LOPCOW weights, execution of TOPSIS analysis, and presentation of results, making it highly suited for solving complex decision-making problems. We have expanded the discussion on future research directions by proposing potential extensions such as integrating machine learning techniques for adaptive weighting and ranking. This could enhance the framework's ability to dynamically adjust criteria weights based on evolving data patterns, improving decision-making accuracy. Additionally, hybridizing the LOPCOW-TOPSIS model with deep learning approaches could further optimize performance in large-scale decision problems.

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